Breaching the Eddington Limit in the Most Massive, Most Luminous Stars

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Massive Stars in “Pillars of Creation”
Massive Stars in the Whirlpool Galaxy
Wind-Blown Bubbles in ISM

Henize 70: LMC SuperBubble

WR wind bubble NGC 2359

Superbubble in the Large Magellanic Cloud
Massive Stars are **Cosmic Engines** & **Cosmic Beacons**

- Help trigger star formation
- Power HII regions
- Explode as SuperNovae
- GRBs from Rotating core collapse Hypernovae
- “First Stars” reionized Universe after BB
Q: So, what **limits** the mass & luminosity of stars?

A: The **force** of light!

- light has momentum, $p=E/c$
- leads to “Radiation Pressure”
- radiation force from **gradient** of $P_{\text{rad}}$
- gradient is from **opacity** of matter
- opacity from both **Continuum** or **Lines**
Continuum opacity from Free Electron Scattering

Thompson Cross Section

$\sigma_{Th} = \frac{8\pi}{3} r_e^2 = \frac{2}{3} \text{ barn} = 0.66 \times 10^{-24} \text{ cm}^2$
The Eddington limit is when the radiative force on free electrons just balances the gravitational force.

**SUPER-Eddington:** radiative force exceeds the force of gravity
Radiative force

\[ g_{rad} = \int_{0}^{\infty} dv \kappa_v F_v / c = \kappa F / c \quad \text{if} \quad \kappa \text{ gray} \]

e.g., compare electron scattering force vs. gravity

\[ g_{el} / g_{grav} = L \sigma_{Th} / 4\pi r^2 c \mu_e \]

\[ \Gamma \equiv g_{el} / g_{grav} = \frac{L}{G M} \frac{\sigma_{Th}}{4\pi r^2 c \mu_e} = \frac{\kappa_e L}{4\pi G M c} \]

• For sun, \( \Gamma_\odot \sim 2 \times 10^{-5} \)
• But for hot-stars with \( L \sim 10^6 L_\odot \); \( M=10-50 M_\odot \)

\[ \Gamma \lessgtr 1 \]
Mass-Luminosity Relation

Hydrostatic equilibrium (\(\Gamma \ll 1\)):

\[
\frac{dP_{\text{gas}}}{dr} = -\rho g \quad \Rightarrow \quad \frac{\rho T}{R} \sim \frac{\rho M}{R^2} \quad \Rightarrow \quad T \sim \frac{M}{R}
\]

Radiative diffusion:

\[
\frac{dP_{\text{rad}}}{d\tau} = \frac{F}{c} \quad \Rightarrow \quad \frac{T^4}{\kappa M / R^2} \sim \frac{L}{R^2} \quad \Rightarrow \quad L \sim \frac{R^4 T^4}{\kappa M}
\]

\[
L \sim \frac{M^3}{\kappa} \quad \Gamma \sim \frac{M^2}{\kappa} \quad (1 - \Gamma)^4
\]
Mass-Luminosity Relation

\[ \log(\Gamma) \]

\[ \log\left(\frac{M}{M_\odot}\right) \]
Key point

- Stars with $M \sim 100 \, M_{\text{sun}}$ have $L \sim 10^6 \, L_{\text{sun}}$ => near Eddington limit!
- Provides natural explanation why we don’t see stars much more luminous (& massive) than this.
Line-Driven Stellar Winds

• Stars near but below the Edd. limit have “stellar winds”

• Driven by line scattering of light by electrons bound to metal ions

• This has some key differences from free electron scattering...
Line Scattering: Bound Electron Resonance

for high Quality Line Resonance, cross section $\gg$ electron scattering

$Q \sim \nu \tau \sim 10^{15} \text{ Hz} \times 10^{-8} \text{ s} \sim 10^7$

$\bar{Q} \sim Z Q \sim 10^{-4} \times 10^7 \sim 10^3$

$\sigma_{\text{lines}} \sim Q \times \sigma_{Th}$

$g_{\text{lines}} \sim 10^3 \times g_{\text{el}}$

$\Gamma_{\text{lines}} \sim 10^3 \times \Gamma_{\text{el}} \gg 1 \quad \{\text{if} \quad L = L_{\text{thin}}\}$
Optically Thick Line-Absorption in an Accelerating Stellar Wind

For strong, optically thick lines:

\[ g_{\text{thick}} \sim \frac{g_{\text{thin}}}{\tau} \sim \frac{1}{\rho} \frac{dv}{dr} \]

\[ \tau \equiv K\rho \frac{V_{th}}{dV/dr} \sim \frac{V_{th}}{V_{\infty}} R_{*} \]

\[ L_{\text{sob}} \ll R_{*} \]
CAK model of steady-state wind

Equation of motion:
\[
\dot{v} \approx -\frac{GM(1 - \Gamma)}{r^2} + \frac{\overline{Q} L}{r^2} \left( \frac{r^2 \dot{v} \dot{v}'}{\dot{M} \overline{Q}} \right)^\alpha
\]

Solve for:
- Mass loss rate
- Velocity law

\[
\dot{M} \approx \frac{L}{c^2} \left( \frac{\overline{Q} \Gamma}{1 - \Gamma} \right)^{1/\alpha - 1}
\]

\[
v(r) \approx v_\infty (1 - R_*/r)^{1/2} \approx v_{esc}
\]

Wind-Momentum Luminosity Law

\[
\dot{M} v_\infty \propto \frac{1}{L^\alpha}
\]

\[
\alpha \approx 0.6
\]
Summary: Key CAK Scaling Results

Mass Flux: \( \dot{m} \sim \frac{F^{1/\alpha}}{g^{1/\alpha-1}} \sim \frac{F^2}{g} \)

Wind Speed: \( V_\infty \sim V_{esc} \sim \sqrt{g} \)

e.g., for \( \alpha = 1/2 \)
How is such a wind affected by (rapid) stellar rotation?
Hydrodynamical Simulations of Wind Compressed Disks

But **caution**: These assume purely **radial** driving of wind
WCD Inhibition

- Stellar oblateness => poleward tilt in radiative flux
- Net **poleward** line-force **inhibits** WCD
Gravity Darkening

increasing stellar rotation
Non-radial line-force + gravity dark.
Effect of gravity darkening on line-driven mass flux

Recall:

\[ \dot{m}(\theta) \sim \frac{F(\theta)^{1/\alpha}}{g_{\text{eff}}(\theta)^{1/\alpha-1}} \sim \frac{F^2(\theta)}{g_{\text{eff}}(\theta)} \]

\( \alpha = 1/2 \)

e.g., for

\( \dot{m}(\theta) \sim F(\theta) \)

highest at pole

w/o gravity darkening, if \( F(\theta) = \text{const.} \)

\( \dot{m}(\theta) \sim \frac{1}{g_{\text{eff}}(\theta)} \)

highest at equator

w/ gravity darkening, if \( F(\theta) \sim g_{\text{eff}}(\theta) \)
Effect of rotation on flow speed

\[ V_\infty(\theta) \sim V_{\text{eff}}(\theta) \sim \sqrt{g_{\text{eff}}(\theta)} \]

\[ g_{\text{eff}}(\theta) \sim 1 - \omega^2 \sin^2 \theta \]

\[ \omega \equiv \Omega / \Omega_{\text{crit}} \]
Eta Car’s Extreme Properties

Present day:

\[ L_{rad} \approx 5 \times 10^6 L_\odot \quad \text{and} \quad \dot{M} \approx 10^{-3} M_\odot / \text{yr} \]
\[ V_\infty \approx 600 \text{ km/s} \]

1840-60 Giant Eruption:

\[ L_{rad} \approx 20 \times 10^6 L_\odot \quad \text{and} \quad \dot{M} \approx 0.5 M_\odot / \text{yr} \]
\[ V_\infty \approx 600 \text{ km/s} \]

\[ \approx L_{kin} = \dot{M} v_\infty^2 / 2 \]

=> Mass loss is energy or “photon-tiring” limited
But lines can’t explain eta Carinae’s mass loss

\[
\dot{M}_{obs} \approx 10^{-3} - 10^{-1} \, M_\odot / \text{yr} \quad \quad \quad V_{obs} \approx 500 - 800 \, \text{km/s}
\]

\[
\dot{M}_{CAK} \approx \frac{L}{c^2} \left( \frac{Q \Gamma}{1 - \Gamma} \right)^{\frac{1}{\alpha} - 1} \quad \quad \quad V_\infty \approx 600 \, \text{km/s} \sqrt[1/2]{M(1 - \Gamma) / R}
\]

\[
\approx 7 \times 10^{-5} \, M_\odot / \text{yr} \quad L_6 \, \overline{Q}_3 \, \Gamma / (1 - \Gamma)
\]

\[
\alpha = 1/2 \quad \quad \quad \quad L_6 \equiv L / 10^6 \, L_{\text{sun}} \quad \quad \quad \overline{Q}_3 \equiv \overline{Q} / 10^3
\]
Super-Eddington
Continuum-Driven Winds

moderated by “porosity”
Porosity

- Same amount of material
- More light gets through
- Less interaction between matter and light

*Incident light*
Monte Carlo
Monte Carlo
Monte Carlo results for eff. opacity vs. density in a porous medium

Log(average density) ~ 1/\rho

“critical density ρ_c

Log(eff. opacity)

blobs opt. thin

blobs opt. thick

~1/ρ
Pure-abs. model for “blob opacity”

\[ \Sigma_{eff} \approx \ell^2 \left[ 1 - e^{-\tau_b} \right] \]

\[ \tau_b \equiv \kappa \rho_b \ell = \kappa \rho \ell / f \]

\[ K_{eff} \equiv \frac{\Sigma_{eff}}{m_b} = K \frac{1 - e^{-\tau_b}}{\tau_b} \approx \frac{K}{\tau_b} ; \tau_b >> 1 \]
Key Point

For optically thick blobs: \[ \kappa_{\text{eff}} \sim \frac{1}{\rho} \]

Blobs become opt. thick for densities above critical density \( \rho_c \), defined by:

\[ \tau_b = \kappa \rho_c \ell / f \equiv 1 \]

\[ \rho_c = \frac{f}{\kappa \ell} \]

volume filling factor
Radiation vs. Gas Pressure

Radiative diffusion

\[ \frac{dP_{\text{rad}}}{d\tau} = \frac{F}{c} \]

Hydrostatic equilibrium

\[ \frac{dP_{\text{tot}}}{d\tau} = \frac{F_c}{c} \]

\[ \frac{P_{\text{rad}}}{P_{\text{tot}}} = \Gamma \quad (\tau \gg 1) \]

\[ \frac{P_{\text{gas}}}{P_{\text{tot}}} = 1 - \Gamma \]
Convective Instability

• Classically expected when $dT/dr > dT/dr_{ad}$
  – e.g., hot-star core $\varepsilon \sim T^{10-20}$; cool star env. $\kappa$ increase

• But $\Gamma(r) \to 1 \Rightarrow$ decreases $dT/dr_{ad} \Rightarrow$ convection
  – e.g., if $\Gamma_e \sim 1/2 \Rightarrow M(r) < M_*/2$ convective!

• For high density interior $\Rightarrow$ convection efficient
  – $L_{\text{conv}} > L_{\text{rad}} - L_{\text{Edd}} \Rightarrow \Gamma_{\text{rad}}(r) < 1$: hydrostatic equilibrium

• Near surface, convection inefficient $\Rightarrow$ super-Eddington
  – but any flow would have $\dot{M} \sim L/a^2$
  – implies wind energy $\dot{M}v_{\text{esc}}^2 \gg L$
  – would “tire” radiation, stagnate outflow
  – suggests highly structured, chaotic surface
Initiating Mass Loss from Layer of Inefficient Convection

\[ F_{\text{conv}} \approx \nu_{\text{conv}} l \frac{dU}{dr} \]
\[ < aH \frac{dP}{dr} \]
\[ \approx a^3 \rho \approx \frac{L}{4\pi R^2} \]

\[ \Rightarrow \dot{M} = 4\pi R^2 \rho a = \frac{L}{a^2} \]

\[ L_{\text{kin}} = \frac{\dot{M}v_{\text{esc}}^2}{2} = \left(\frac{v_{\text{esc}}^2}{2a^2}\right) L \gg L \]

\[ \Rightarrow \text{flow would stagnate due to “photon tiring”} \]
Stagnation of photon-tired outflow

\[ \Gamma_*(x) = 1 + \sqrt{x} \]

\[ m \equiv \frac{\dot{M} V^2}{2 L_*} \]

\[ x = 1 - \frac{R_*}{r} \]
Shaviv
2001
A POROSITY-LENGTH FORMALISM FOR PHOTON-TIRING-LIMITED
MASS LOSS FROM STARS ABOVE THE EDDINGTON LIMIT

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ABSTRACT

We examine radiatively driven mass loss from stars near and above the Eddington limit. Building
upon the standard CAK theory of driving by scattering in an ensemble of lines with a power-law
distribution of opacity, we find an additional factor of radiative driving.

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Porous opacity

\[ \tau_b = \kappa \rho_b l = \kappa \rho \frac{l}{l^3/L^3} \equiv \kappa \rho h \]  

“porosity length”

\[ \tau_b \gg 1 \quad ; \quad \Gamma_{\text{eff}} \approx \frac{\Gamma}{\tau_b} = \Gamma \frac{\rho_c}{\rho} \]

\[ \rho_c = 1/\kappa h \]
Super-Eddington Wind

Wind driven by continuum opacity in a porous medium when $\Gamma_* > 1$

At sonic point: 

$$ \Gamma_{\text{eff}}(r_S) = \Gamma \frac{\rho_c}{\rho_S} \equiv 1 $$

$$ \rho_S = \Gamma \rho_c = \Gamma_* / \kappa h $$

$$ h \approx \frac{H}{a^2 / g_*} $$

"porosity-length ansatz"

$$ \dot{M} = 4\pi R_*^2 \rho_S a $$

$$ \approx \frac{L_*}{ac} $$

$$ \approx 0.001 \frac{M_\odot}{\text{yr}} \frac{L_6}{a_{20}} $$
Power-law porosity

At sonic point: \[ \Gamma_{\text{eff}}(r_S) = \Gamma \left( \frac{\rho_c}{\rho_S} \right)^{\alpha} \equiv 1 \]

\[ \dot{M} = 4 \pi R^2 \rho_s a \approx \frac{L_*}{ac} \Gamma^{-1+1/\alpha} \]

\[ \dot{M}_{\text{CAK}} \approx \frac{L_*}{c^2} \left( \frac{\overline{Q \Gamma}}{}\right)^{-1+1/\alpha} \]
Results for Power-law porosity model
Effect of gravity darkening on porosity-moderated mass flux

\[ \dot{m} = \frac{\dot{M}}{4\pi R^2} \quad \dot{m}(\theta) \sim F(\theta) \left( \frac{F(\theta)}{g_{\text{eff}}(\theta)} \right)^{-1+1/\alpha} \]

w/ gravity darkening, if \( F(\theta) \sim g_{\text{eff}}(\theta) \)

\[ \dot{m}(\theta) \sim F(\theta) \quad \text{highest at pole} \]

\[ v_\infty(\theta) \sim v_{\text{eff}}(\theta) \sim \sqrt{g_{\text{eff}}(\theta)} \quad \text{highest at pole} \]
Eta Carinae
Summary Themes

• Continuum vs. Line driving

• Prolate vs. Oblate mass loss

• Porous vs. Smooth medium
Future Work

• Radiation hydro simulations of porous driving

• Cause of \( L > L_{\text{Edd}} \)?
  – Interior vs. envelope; energy source

• Cause of rapid rotation
  – Angular momentum loss/gain

• Implications for:
  – Collapse of rotating core, Gamma Ray Bursts
  – Low-metallicity mass loss, First Stars