PHYS133 – Lab 1
Math Review

Goal:
- To review mathematical concepts that will be used in this course.

What You Turn In:
- The worksheet in this manual.

Background:
This course requires the use of several concepts from high school algebra that you may not have seen or used in many years. This worksheet has been designed to help remind you how to round, use scientific notation, and write answers in significant figures, as well as to cover math concepts such as exponent rules, conversion factors, and basic trigonometry.

Procedure:
1. Rounding
In general, rounding rules are straightforward:
   - For numbers 0-4: round down.
   - For numbers 5-9, round up.

   You need to know the place to which you are rounding. Take for example the number 123,456.7890

   If we want to round our number to the thousands place, we check the number in the hundreds place, here a 4. According to the rounding rules, 4 gets rounded down, so we don’t change the number 3 in the thousands place. We truncate the rest of the number at the thousands place, and fill in zeros before the decimal so that the thousands place retains its value. Our number becomes 123,000.

   If we want to round our number to the tenths place, we look at the number in the hundredths place, an eight. The rounding rules above state that we round eights up, and so the 7 in the tenths place becomes an 8. The value is given as 123, 456.8

2. Rules for dealing with exponents
A simplistic way of dealing with an exponent is to say it is the number of times a number is multiplied by itself. This works well for positive integers:

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1 The “Exponent Rules” and “Scientific Notation” portions of this worksheet are closely adapted from Appendix C of Bennett, Jeffrey O., Megan Donahue, Nicholas Schneider, and Mark Voit. The Essential Cosmic Perspective. 4th ed. San Francisco: Pearson/Addison Wesley, 2008. Print.
$2^3 = 2 \times 2 \times 2 = 8$

But we can also talk about negative exponents where any number to the -1 power is the number’s multiplicative inverse. So, you just take “one-over” the number to the positive exponent:

$$3^{-1} = \frac{1}{3} \quad 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

The following rules apply when multiplying numbers with exponents:

$$f^a g^a = (fg)^a \quad \text{example:} \quad 5^2 6^2 = (5 \times 6)^2$$
$$f^a f^b = f^{a+b} \quad \text{example:} \quad 3^2 3^4 = 3^6 \quad \text{or} \quad 5^3 5^{-2} = 5^1$$

Exponents that are not integers:

$$f^{1/a} \quad \text{example:} \quad 3^{1/2} = \sqrt{3} \quad \text{or} \quad 3^{1/4} = \sqrt[4]{3}$$
$$f^{a/b} = \sqrt[b]{f^a} = (\sqrt[b]{f})^a \quad \text{example:} \quad 4^{6/2} = \sqrt[2]{4^6} = (\sqrt[2]{4})^6 = 4^3$$

Multiplication rules are the same as those for integers. Your calculator can handle decimal exponents quite easily, but problems can often be simplified by knowing these rules.

### 3. Scientific Notation – Powers of Ten

A very useful way to deal with very large or small numbers is to use scientific notation. This uses integer exponents of 10 to make writing large or small numbers more concisely.

For example, 100 billion would be

$$100,000,000,000 = 10^{11}$$

and

$$0.0000000001 = 10^{-9}$$

Such that we can express large values such as the distance to the moon, 384,000,000 m as

$$384,399,000 \text{ m} = 3.84399 \times 10^8 \text{ m}$$

If we were asked to round this to three digits, we would have 3.84 x 10$^8$ m.

Often, it is impractical to even write out leading or trailing zeros such as the mass of an hydrogen atom (1.67 x 10$^{-27}$ kg) or the energy produced by the sun (3.828 x 10$^{26}$ Watts).

**Note that there is one and only one digit to the left of the decimal point and it must always be non-zero to be in scientific notation.**

It is customary to use prefixes for certain powers of 10. Common ones you should know are
### Significant Digits

The number of significant digits in a value represents how well that value is known. In reality, we use something called *error values* to show how accurate a value is, but unless a value is given, the accuracy is determined by the right-most non-zero number. That is, if you are told the distance from the University of Delaware to Kitt Peak National Observatory is 2,300 miles there are two significant digits (23). The zeroes to the right of the last non-zero digit are ignored.

Likewise, any non-zero digit to the left of the first non-zero digit is also ignored. A mass of 0.00267 g would have three significant digits (the 267).

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**When adding and subtracting** values, the value with the *least significant digit* in the *left-most* position is used. For example,

\[
195.68 \\
+ 27.4 \\
+ 0.254 \\
\hline
223.334
\]

Since the number with the left-most significant digit is 27.4, we make the significant digit in the result at the same place (in this case the tenths place). Our final value to significant digits is 223.3. Note that we round, not truncate.

**When multiplying or dividing** values, the value with the *least number of significant digits* is used to give the number of significant digits in the result. For example,

\[
254.8 \times 2.1 \times 1.648 = 881.81184 \quad \Rightarrow \quad 880 \text{ with significant digits}
\]
\[
\frac{856.2}{12.8} = 66.890625 \quad \Rightarrow \quad 66.9 \text{ with significant digits}
\]

Since the number with the left-most significant digit is 27.4, we make the significant digit in the result at the same place (in this case the tenths place). Our final value to significant digits is 223.3. Note that we round, not truncate.

5. Units and Conversions

Almost everything we measure needs to have some unit involved. The units we use to measure time, distance, mass, etc. have historically been based on sizes of body parts, steps or other easy items at hand. Over the millennia, these units have become more standardized to natural phenomena, although their origins were long ago based on the rotation of the Earth, etc.

Time

The second is the standard unit of time. Now, it is based on a particular number of vibrations of a specific isotope of Cesium. This was due to the second being based on the Babylonian system of dividing a solar day into 24 hours, each hour into sixty minutes and each minute into sixty seconds. This would make one second 1/86400 of a day. Due to the Earth’s rotation slowing and other factors that is not exact enough so we have an agreed upon length of time for a second that we all use. This is why you occasionally hear about a leap second being added to a day.

Length

The meter has a similar history, it was originally set to be 1/10,000,000 the distance from the equator to the North Pole. This was changed to be the particular length of a platinum (later platinum-iridium) bar kept in Paris. All other meter rods were set to check against this. Since 1983 it has been defined as the distance light travels in a vacuum in exactly 1/299,792,458 of a second. This is due to our understanding that the speed of light is a physical constant and if we know how long a second is, we can find the length of a meter by using light.

Mass

The kilogram has a similar history to the meter, but it has yet to be defined in terms of actual physical constants. Its definition is the mass of a particular “International Prototype Kilogram” kept in Paris. Several natural constant definitions have proposed, but not yet adopted.

There are other basic measurement that we make (e.g., electrical currents, temperature, luminous intensity) and you will discover some of them later in this course.

Converting from one unit to another is critical. In many formulae that deal with different kinds of values to get another. An example that you will come across is Kepler’s third law which relates the period of a planet/moon/satellite, its distance from what it’s orbiting and the object’s mass and the mass of the object it’s orbiting. In the equation, there is a constant that allows us to relate these values, but only if the values use particular units.

Since we can measure these units in many ways, we need to convert between them. The first thing we need is how much of one unit equals another. There are plenty of examples:
The way to use these is by remembering that anything multiplied by one is the same thing. The trick is finding the right form of one.

**Example:**

How many feet are in 1.000 kilometer?

\[
1 \text{ km} = \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{100 \text{ cm}}{1 \text{ m}} \cdot \frac{1 \text{ in}}{2.54 \text{ cm}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 3281
\]

Note that each value in parentheses equals one. Also, note that the appropriate units are in the denominator to cancel out the units in the numerator from which we want to convert. You use conversion factors until you reach the desired unit(s).

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6. **Elementary Trigonometry and Geometry**

You should be able to use sine, cosine, and tangent. These will be very useful in working out distances later. The definition of these basic trigonometric functions is

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

You should also remember the Pythagorean theorem, \((\text{adjacent})^2 + (\text{opposite})^2 = (\text{hypotenuse})^2\).

The angles will often be measured in degrees (or portions of degrees). Make sure your calculator is in the correct *mode*.

**Radians will be introduced in a later lab/lecture.**

You are also expected to know basic geometry. Such things will be the area and circumference of a circle based on its radius.
Arithmetic

Find the answers to the following problems, rounding to the correct significant digit.

1. 1895.376 + 709.2353

2. 5467.2 – 2677.89

3. 1024 ÷ 800

4. 6285 x 2.00

5. \( \frac{18.6}{0.5255} \)

6. 5.0 x 9824.66

7. 365.25 x 24.0

8. 16.6^2
Scientific Notation

Find the answers to the following problems, rounding to the correct significant digit and expressing the results in scientific notation.

1. \((6.26 \times 10^3)(5.7 \times 10^{-4})\)

2. \(\frac{(6.26 \times 10^3)^3}{(5280)^2}\)

3. \((2.354 \times 10^6)^{\frac{1}{2}}(2.354 \times 10^6)^2\)

4. \((25)^{\frac{1}{2}}(2)^2\)

5. \(\frac{1.00 \times 10^4}{1.00 \times 10^9}\)

6. \(\frac{(6.00)^3(2.05)^2}{(9.0)^{\frac{1}{2}}}\)
Prefixes
Write each value with the correct prefix. For example, 90 thousand meters would be 90 km. Do not use scientific notation here. (lightyear = ly, parsec = pc)

1. 13 billion years
2. 91 million miles
3. 200 thousand meters
4. 3.26 lightyears
5. 12 milliseconds
6. 6536 nanometers
7. 23 megaparsecs
8. 324 centimeters
9. 75 microseconds
Using only the conversion factors found in this manual convert the following:

1. The average distance from the Earth to the Sun is 1.00 AU. Convert the radius of the sun, 696,000 km, into astronomical units (AU)

2. If the mass of the Earth is $5.972 \times 10^{24}$ kg, what is its mass in solar masses $M_\odot$?

3. The nearest star is 4.3 ly distant. How many AU (distances from Earth to the Sun) is this?

4. The age of the universe is about 13.7 billion years. How long is this in seconds (use scientific notation for your answer)
Solve the following:

1. Kepler’s third law is given as $p^2 = \frac{a^3}{M}$, where $p$ is the period of orbit, $a$ is the radius of the orbit and $M$ is the mass of the object being orbited. Solve this equation for the mass ($M$).

2. The temperature ($T$) of an object can be determined by the wavelength of light ($\lambda$) of maximum intensity. The relationship is given as $\lambda = \frac{b}{T}$. Find $b$ if the Temperature of an object is 5270 K (kelvin) and the wavelength of maximum intensity observed is 550 nm. Express this value in $\mu$m-K.

3. Solve the following equation for $x$: $x^2 - 6x + 5 = 0$
4. On the summer solstice at local noon, a 1.00 m tall stick extends a shadow of 25 cm. What angle above the horizon is the sun?

5. Continuing from the previous question, if the Earth’s circumference is about 40,000 km, how far south do need to travel to find a stick that casts no shadow (show your work).
What have I learned in this laboratory session?

What improvements could be made to this lab?

On the whole, this lab was (circle your choice):

- Too long
- Not long enough
- The appropriate length of time

If you selected “too long”, what would you shorten to make it an appropriate length of time?