Phys133 Final Exam
Short problem samples

1. A spectral line with rest wavelength $\lambda=500$ nm is observed at a wavelength $\lambda=550$ nm in the spectrum of a distant galaxy.

   a. What is the galaxy’s redshift $z$?
      $z = \frac{\Delta \lambda}{\lambda} = \frac{(550-500)}{500} = 0.1$

   b. Is this galaxy moving toward or away from us?
      $z > 0 \Rightarrow$ redshift $\Rightarrow$ away

   c. What is the galaxy’s radial velocity $V$ (in km/s)?
      $V = z \cdot c = 0.1 \cdot c = 30,000$ km/s

   d. If this redshift is due to the universe’s expansion with a Hubble constant $H_0=60$ km/s/Mpc, what is the distance (in Mpc) of this galaxy?
      $d = \frac{V}{H_0} = \frac{30,000}{60} = 500$ Mpc

   e. How long ago (in millions of years) did the light we observe leave this galaxy?
      $t = \frac{d}{c} = \frac{500}{c}$ Mpc/c $= 3.26 \times 500$ Mly/c $= 1.63$ Gyr

2. Galaxies A and B have redshifts with ratio $z_A / z_B = 9$. Assuming the redshifts are due entirely to the universe’s expansion:

   a. What is the associated ratio of their radial speeds, $V_A / V_B$?
      $V = z \cdot c \Rightarrow \frac{V_A}{V_B} = \frac{z_A}{z_B} = 9$

   b. What is the associated ratio of their distances, $d_A / d_B$?
      $d = \frac{V}{H_0} \Rightarrow \frac{d_A}{d_B} = \frac{V_A}{V_B} = \frac{z_A}{z_B} = 9$
c. If they both have the same physical size, what is the ratio of their angular size, $a_A/a_B$?
$$a = s/d, \quad s_A = s_B \quad \Rightarrow \quad a_A/a_B = d_B/d_A = 1/9$$

d. If they both have the same apparent brightness, what is the ratio of their associated luminosity, $L_A/L_B$?
$$F = L/4\pi d^2, \quad F_A = F_B \quad \Rightarrow \quad L_A/L_B = (d_A/d_B)^2 = (9)^2 = 81$$

3. The Cosmic Microwave Background (CMB) is observed to have its peak brightness at a wavelength of about $\lambda_{\text{max}} = 1$ mm.

a. What is the associated blackbody temperature of the current day universe (in K)?
$$T = 6000 \text{ K} \left( 0.5 \ \mu\text{m} / \lambda_{\text{max}} \right) = 6000 \text{ K} \left( 0.5 \ \mu\text{m} / 1 \ \text{mm} \right) = 3 \text{ K}$$

b. Assuming that this radiation was emitted from an epoch at redshift $z_{\text{CMB}}=1000$, what was the temperature (in K) at the time when this CMB radiation was emitted.
$$\lambda_{\text{max,em}} = \lambda_{\text{max}} / z_{\text{CMB}} = 1 \ \text{mm} / 1000 = 1 \ \mu\text{m}$$
$$T_{\text{em}} = 6000 \text{ K} \left( 0.5 \ \mu\text{m} / \lambda_{\text{max,em}} \right) = 6000 \text{ K} \left( 0.5 / 1 \right) = 3000 \text{ K}$$

c. What happened at this time that allowed these CMB photons to be released?
Electrons combined with protons to make neutral Hydrogen, meaning the electrons no longer scatter the light, allowing it to propagate through universe (redshifting with the expansion)
4. Slit spectra along an edge-on galaxy show a line of rest wavelength \( \lambda = 500 \) nm to be at an observed wavelength \( \lambda_{\text{obs}} = 500.5 \) nm for all positions beyond a distance \( R_o \) to the east side of the center, but at \( \lambda_{\text{obs}} = 499.5 \) nm for positions more than \( R_o \) to the west side?

   a. What is the rotation speed \( V \) associated with these shifts?
   \[ V = c \frac{\Delta \lambda}{\lambda} = c \frac{(500.5 - 500)}{500} = c \frac{0.5}{500} \]
   \[ = c \frac{1}{1000} = 300 \text{ km/s} \]

   b. Which side of the galaxy (east or west) is coming toward us, and which is receding away from us?
   - East \( \lambda_{\text{obs}} > \lambda \) => redshift => away
   - West \( \lambda_{\text{obs}} < \lambda \) => blueshift => toward

   c. For distances \( R \geq R_o \) from the galaxy center, what is the ratio \( M(R)/M(R_o) \) of the mass within radius \( R \) compared to that within radius \( R_o \)?
   \[ V^2/R = GM/R^2 \Rightarrow M(R)/M(R_o) = \left( \frac{V(R)}{V(R_o)} \right)^2 \frac{R}{R_o} \]
   \[ V(R) = V(R_o) \Rightarrow M(R)/M(R_o) = R/R_o \]

   d. If the masses were instead all contained within \( R_o \), what would be the speed ratio \( V(4R_o)/V(R_o) \) at radius \( R=4R_o \) compared to that at radius \( R_o \)?
   \[ V^2/R = GM/R^2 \Rightarrow V(4R_o)/V(R_o) = \sqrt[4]{R_o/4R_o} = \sqrt[4]{1/4} = 1/2 \]
5. In the figure, which model(s) correspond to a universe that is 
   a. Re-collapsing D
   b. Accelerating  A
   c. Coasting     B
   d. Critical     C
   e. Substantially younger than one Hubble time $t_H$   C & D

6. What is the age (in Gyr) of a coasting universe with Hubble constant
   \( H_0 = \frac{1}{(100 \text{ (km/s)/Mpc}) = 10 \text{ Gyr}} \)
   a. 50 (km/s)/Mpc?  10 Gyr $100/50 = 20$ Gyr
   b. 70 (km/s)/Mpc?  10 Gyr $100/70 = 14.3$ Gyr
   c. 100 (km/s)/Mpc? 10 Gyr $100/100 = 10$ Gyr
   d. Which of these is compatible (or not) with the inferred ages of globular clusters?
      Oldest globular clusters about 13 Gyr, so
      A & B compatible, C not compatible
7. Suppose that over a period of 10 years we observe a star make a complete orbit at a fixed angular radius of 1/8 arcsec about the galactic center.

a. Given that the galactic center is about 8 kpc from earth, what is the physical radius \( r \) (in AU) of the star’s orbit?
\[
\frac{r}{\text{AU}} = \frac{a}{\text{arcsec}} \frac{d}{\text{pc}} = \frac{1}{8} \times 8000 = 1000
\]

b. What do these observations imply for the mass \( M \) (in \( M_{\odot} \)) of the central black hole?
\[
\frac{M}{M_{\odot}} = \left(\frac{r}{\text{AU}}\right)^3 \left(\frac{P}{\text{yr}}\right)^2 = 1000^3/10^2 = 10^7
\]

c. How does this compare to estimates for the actual mass of the black hole at the center of our Galaxy.
Actual black hole mass is about 3 Million \( M_{\odot} \), so this is about factor 3 higher

8. The Andromeda galaxy is at a distance of about 0.8 Mpc. Another galaxy has Cepheid variables that have an apparent brightness that are 1/10,000 the brightness of similar period Cepheids in Andromeda.

a. What is the distance \( d \) (in Mpc) of this other galaxy?
\[
d = \sqrt{\frac{L}{4\pi F}} \Rightarrow d = 0.8 \text{ Mpc} \times \frac{1}{\sqrt{1/10,000}} = 80 \text{ Mpc}
\]

b. If this galaxy has an observed redshift of \( z=0.02 \), what is its radial velocity \( V \) (in km/s)?
\[
V = z \times c = 0.02 \times c = 6000 \text{ km/s}
\]

c. What is the associated value of the Hubble constant \( H_0 \) (in (km/s)/Mpc)?
\[
H_0 = \frac{V}{d} = \frac{6000}{80} \text{ (km/s)/Mpc} = 75 \text{ (km/s)/Mpc}
\]

d. What is the associated Hubble time \( t_H \) (in Gyrs)?
\[
t_H = 10 \text{ Gyr} \times \frac{100}{75} = 13.3 \text{ Gyr}
\]