 PHYS-333: Problem set #7

Please write neatly and show your work. Please put a box around all your final answers. Answers without a box may not receive credit. Big magnitude answers should be given in scientific notation. Answers generally need only have 2 or 3 significant figures. At the top of your front page, near your own name, please list the name of all persons with whom you consulted in solving the problems.

1. Gravitational Lensing
   Suppose a distant galaxy cluster with redshift \( z = 0.2 \) has two identical quasar images at equal angles \( \alpha = 10 \) arcsec on each side of the cluster center.
   a. Assuming a Hubble constant \( H_0 = 67 \) (km/s)/Mpc, what is the distance \( d \) (in Mpc) to the lensing galaxy?
   b. Use this distance \( d \) and the angle \( \alpha \) to estimate the closest distance \( b \) (in kpc) that the quasar’s light passes to the center of the galaxy.
   c. Using this \( b \), and relating \( \alpha \) to the \( \theta \) in Einstein’s gravitational lensing formula, estimate the mass \( M \) (in \( M_\odot \)) of the lensing galaxy cluster. You may assume here that the quasar source is very much further away than the lensing cluster.

2. Gravitational redshift as alternative explanation of quasar redshift
   Suppose we try to explain the redshift of 3C273 (\( z = 0.158 \)) as a gravitational redshift, rather than being from cosmological expansion.
   a. Relative to Schwarzschild radius \( R_s \), from what radius \( R_o \) is the radiation emitted?
   b. If the width of lines is 0.1% of their central wavelength, what is the range of radii (relative to \( R_s \)) from which the radiation can be emitted.
   c. For a more physically reasonable assumption that any emission would come from at least radius range \( \pm 10\% \) around the central radius \( R_o \), what would be the relative width \( \Delta \lambda/\lambda_o \) of the observed emission line?
3. **Critical Universe Redshift**

Consider a critical universe $\Omega_m = 1$ without dark energy ($\Omega_\Lambda = 0$) but with a local Hubble constant equal to $H_o \approx 67 \text{ (km/s)/Mpc}$.

a. Derive a formula for redshift $z(d)$ vs. distance $d$ (in Gly).

b. Show that for small distances $d \ll c/H_o$, this recovers the simple linear Hubble law $cz = H_o d$.

c. Compute the time since the Big Bang, in Gyr.

d. Compare this time to the age of a Globular cluster with a main-sequence turnoff at luminosity $L_{to} = 0.75 L_\odot$.

e. What does this say about the viability of this as a model for our universe? What about closed-universe models with $\Omega_m > 1$? (Assume the above Hubble constant measurement is accurate, and that there is no dark energy.)

4. **Empty Universe**

Next consider the case of an effectively “empty” universe with $\Omega_m = \Omega_\Lambda = 0$, that is again expanding with a locally measured Hubble constant $H_o \approx 67 \text{ (km/s)/Mpc}$.

a-d. Repeat parts a-d of previous problem for this case of an empty universe.

e. What does the result in part d here say about the formal viability of this as a model for our universe?
5. **Empty vs. Critical Universe**

a. For the empty universe model of previous problem, invert the formula for \( z(d) \) to derive an expression for distance as a function of redshift \( z \). For this use the notation \( d_0(z) \), where the subscript “0” denotes the null value of \( \Omega_m \).

b. If a distance measurement is accurate to 10%, at what minimum redshift \( z_o \) can one observationally distinguish the redshift vs. distance of an empty universe from a strictly linear Hubble law \( d = cz/H_o \). (*Hint:* At what redshift do the two distance laws differ by 10%?)

c. Using the results from previous problems, now derive an analogous distance vs. redshift formula \( d_1(z) \) for the critical universe with \( \Omega_m = 1 \) (and \( \Omega_\Lambda = 0 \)).

d. Again if a distance measurement is accurate to 10%, at what minimum redshift \( z_1 \) can one observationally distinguish the redshift vs. distance of such a critical universe from a strictly linear Hubble law.

e. Finally, again with a distance measurement accurate to 10%, at what minimum redshift \( z_{10} \) can one observationally distinguish the redshift vs. distance of a critical universe from an empty universe?

f. For the redshift \( z_{10} \) in part e, what are the corresponding distances (in Mpc) in the empty \( (d_0) \) and critical universes \( (d_1) \)?