angular size (a), physical size (s), distance (d), parallax (p); solid angle (Omega)
\[ a = \frac{s}{d} ; \quad a/\text{arcsec} = (s/\text{AU})/(d/\text{pc}) \]
\[ \frac{d}{\text{pc}} = \frac{1}{(p/\text{arcsec})} \]
\[ \Omega \sim a^2 \sim 1/d^2 \]

flux F (apparent brightness), luminosity L (absolute brightness), intensity I (surface brightness)
\[ F = \frac{L}{4 \pi d^2} \]
\[ I \sim \frac{F}{\Omega} \Rightarrow \text{independent of } d \]

apparent magnitude m, absolute magnitude M, distance modulus m-M
\[ m_1 - m_2 = 2.5 \log(F_2/F_1) \]
\[ M = 5 - 2.5 \log(L/L_{\odot}) \]
\[ m - M = 5 \log(D/10 \text{ pc}) \]

standard candle method for distance
\[ d = \sqrt{\frac{L}{4 \pi F}} \]

light: wavelength (\lambda), frequency (\nu), speed (c) photon energy (E)
\[ \lambda \nu = c \]
\[ E = \hbar \nu \]

Black-body
- color (peak wavelength) vs. temperature
- surface flux vs. temperature

\[ \lambda_{\max} \approx 500 \text{ nm } (T_{\sun}/T) \]
\[ F_{\text{surf}} = \frac{L}{4 \pi R^2} = \sigma T^4 \] (\sigma = Stefan-Boltzmann constant)
\[ L = F_{\text{surf}} \times 4 \pi R^2 \Rightarrow \frac{L}{L_{\sun}} = (T/T_{\sun})^4 \left(\frac{R}{R_{\sun}}\right)^2 \]

stellar absorption lines; spectral type \( \Rightarrow \) temperature

**OBAFGKM (hot => cool)** (Oh Be a Fine Girl/Guy Kiss Me)
Sun is G2 star, Teff=5800 K \( \sim \) 6000 K

Bohr atom
- energy levels, in eV and equivalent wavelength
- orbital radii, Bohr radius
- emission vs. absorption

\[ E_n = -13.6 \text{ ev/n}^2 \]
\[ \lambda_n = \frac{\hbar c}{E_n} = 1240 \text{ nm } (\text{ev/E}) \sim 1 \text{ micron } (\text{ev/E}) \]
\[ r_n = n^2 r_1 ; \quad r_1 = \text{Bohr radius} = 0.5 \text{ Angstrom} = 5 \text{ nm} \]

Space velocity:
Proper motion $\Rightarrow$ transverse speed
Doppler Effect $\Rightarrow$ speed toward/away

$$V_t = 4.7 \frac{\mu}{p} \text{ km/s} \quad \mu \text{ in arcsec/yr} \quad p=\text{parallax in arcsec}$$

$$V_r = c \frac{\Delta \lambda}{\lambda}; \quad \Delta \lambda = \lambda_{\text{obs}} - \lambda$$

Newton's law of gravity:
- Surface gravity
- $V_{esc}, V_{orb}$
- Virial theorem: $E_{kin} = -E_{grav}/2$

$$F_g = G \frac{M_1 M_2}{r^2}$$
$$g = \frac{GM}{R^2}$$
$$V_{orb}(r) = \sqrt{\frac{GM}{r}}$$
$$V_{esc} = \sqrt{2} V_{orb}(r=R)$$

Binaries
- Visual: period $P$, radius $a$, $\Rightarrow$ total stellar mass $M_1+M_2$
- Spectroscopic: Doppler shift $\Rightarrow$ speed + period $\Rightarrow$ stellar mass
- Eclipsing $\Rightarrow$ stellar radii

$$(M_1+M_2)/M_{\odot} = (a/AU)^3/(P/\text{yr})^2$$

For planet around much more massive star ($M_1 = M \gg M_2=\text{planet}$)

$$M/M_{\odot} = (a/AU)^3/(P/\text{yr})^2$$
$$= (V/V_e)^3 (P/\text{yr})$$
$$= (V/V_e)^2 (a/AU)$$

where $V_e=2 \pi \text{ AU/yr} = 30 \text{ km/s} = \text{earth's orbital speed around sun}$

Rotation from Doppler broadening of absorption lines with wavelength width $\Delta \lambda$

$$V_{rot} = c \frac{\Delta \lambda}{\lambda}$$

Light absorption & emission:
- cross section $\sigma$, opacity $\kappa$, mean free path $l$, optical depth $\tau$
- exponential attenuation of light by absorption
- emission + absorption: Eddington-Barbier relation
- Radiative diffusion in interior
- Random walk escape of light

$$\kappa = \frac{\sigma}{m} \quad \rho = n \cdot m$$
$$mfp = \frac{1}{(\sigma \cdot n)} = \frac{1}{(\kappa \cdot \rho)}$$
$$\tau(s) \sim \int ds/mfp \sim \int \kappa \rho ds \quad s=\text{line of sight distance}$$
$$I = I_0 \exp[-\tau]$$
$$I_{obs} = B(\tau=1); \quad \tau(s) = \mu \tau(r); \quad B \sim T^4$$
$$N_{scat} \sim \tau^2 \Rightarrow \text{diffusion time} = R \tau/c$$

Atomic Opacity: electron scattering, free-free, bound-free

$$\kappa_{es} = 0.2 \ (1+X) = 0.34 \text{ cm}^2/\text{g} \quad \text{for solar H-frac X=0.7}$$
Dust extinction, reddening, **extinction magnitude vs. optical depth**
- extinction mag \( A \sim \tau \)
  - \( \sigma \sim 1/\lambda \) \( \Rightarrow \) \( A_{\text{red}} < A_{\text{blue}} \Rightarrow \) reddening

**HR-Diagram:**
- color (or spectral type) vs (abs) magnitude
- temperature vs. luminosity
- main sequence \( \Rightarrow \) Hydrogen fusion in core
- red giants, white dwarfs \( \Rightarrow \) post-main-sequence

**Stellar structure:**
1. **Hydrostatic Equilibrium** (pressure gradient vs. gravity)
2. **Radiative Diffusion** (flux vs. temperature gradient)
   - with ideal gas law \( P=\rho kT/\mu \)
   - \( 1+2 \Rightarrow \) mass-luminosity relation

\[
\frac{dP}{dr} = -\rho GM/r^2 \\
\frac{dP_{\text{rad}}}{d\tau} = F/c \\
Prad \sim B \sim T^4 \Rightarrow F \sim dB/d\tau \sim -(T^3/kappa \rho) dT/dr \\
(T/Teff)^4 = (3/4)(\tau+2/3)
\]

\( L/L_{\odot} \sim (M/M_{\odot})^3 \) for stars with radiative envelopes, e.g. on main sequence, horizontal track

But above breaks down when envelopes develop **Convection**
- \( |dT/dr|_{\text{rad}} > |dT/dr|_{\text{ad}} \Rightarrow \) convective energy transport (instead of radiative diffusion)
- occurs when H recombines \( \Rightarrow \) \( \kappa_{\text{bf}} \Rightarrow |dT/dr|_{\text{rad}} \sim \kappa_{\text{ad}} \)
- enforces \( dT/dr = dT/dr(\text{ad}) \)
- convection much more efficient at transporting heat to surface \( \Rightarrow \) allows much higher \( L \)

**Hayashi track** \( (T_{\text{surf}} \sim 3000 \text{ K}) \) have convective envelopes

**Energy sources**
- Chemical reactions too inefficient \( \Rightarrow \) last only few \( 10^4 \) years
- Gravitational energy \( \Rightarrow \) last "**Kelvin Helmholtz**" (KH) time \( \sim 30 \text{ Myr} \)
- H-fusion: efficiency \( \Rightarrow \) Chemical burning

\[
\text{eff(chem)} \sim \text{ev/Gev} \sim 10^{-9} \Rightarrow \text{tchem} \sim 10^3 \text{ yrs for sun} \\
\text{eff(4H->He)} \sim 0.007 = 0.7 \% \text{ of mass} \Rightarrow \text{energy}
\]

H-fusion requires proton thermal speed \( v_{\text{th}} \sim 690 \text{ km/s} \sim V_{\text{esc}} \)
- \( T_{\text{core}} \sim 15 \text{ MK} \)

**Stellar mass range:** \( 0.08 \text{ Msun} < M \sim< 200 \text{ Msun} \)
- \( M < 0.08 \text{ Msun} \Rightarrow \) **Brown dwarf**: electron degeneracy keeps core temperature below nuclear fusion \( T_{\text{fusion}} \sim< 15 \text{ MK} \)
- \( M >200 \text{ Msun} \Rightarrow \) Eddington limit: electron scattering of stellar luminosity \( \Rightarrow \) radiation force > gravity
Evolution

Pre-MS: Hayashi track => Kelvin-Helmholtz (KH): gravitational contraction
Main sequence (MS) (H-core burning) lifetime
Cluster HRD, Main sequence turnoff => age

\[ t_{KH} \approx \frac{GM^2}{R}/L \approx 30 \text{ Myrs for sun} \]
\[ t_H \approx \text{eff} \times 0.1 \frac{M}{c^2}/L \approx 10 \text{ Byr for sun}, \text{ with eff}(4H\rightarrow\text{He}) \approx 0.7\% \]

\[ t_{\text{ms}} \approx \frac{M}{L} \approx 1/M^2 \Rightarrow t_{\text{ms}} \approx 10 \text{ Byr} (M_{\text{sun}}/M)^2 \]
for cluster: \( t_{\text{age}} \approx 10 \text{ Byr} (L_{\text{sun}}/L_{\text{turnoff}})^{(2/3)} \)

\textbf{Sun; MS} => core H exhaustion => H-shell burning => overluminous => inv. KH
expansion
=> convective envelope, climb Hayashi track to tip of Red giant branch (RGB) => He-flash

=> Horizontal Branch (HB) w/ \( 3\text{He} \rightarrow C \) => He plus \( 4\text{H0} \rightarrow \text{He shell burning} \)

Asymptotic Giant Branch (AGB)
=> Planetary Nebula (PN) => CO White Dwarf (WD)

Above also roughly holds for all stars with initial mass \( M \leq 8 \text{ Msun} \)

\textbf{High-mass (M>~ 8\text{Msun})} => core H exhaustion => H-shell burning => inv. KH exp.
=> He core + H-shell => multiple shell burning up to Fe in core with diminishing efficiency

\textbf{Binding energy/nucleon vs. atomic #}
steep jump from H to He (7 Mev)
but then levels off
minimum at Fe (iron)
fusion up to Fe => releases energy
fusion beyond Fe => requires input energy

\textbf{Stellar death}
As a function initial stellar mass \( M_i \):

\( M_i < 8\text{Msun} \Rightarrow \) Planetary Nebula => White Dwarf
White Dwarf mass limit: \( M_{\text{wd}} < M_{\text{chandra}} = 1.4 \text{ Msun} \)

\( M_i > 8\text{Msun} \Rightarrow \) core collapse SuperNova (SN)

\( 8 \text{ Msun} < M_i < 30 \text{ Msun} \Rightarrow M_{\text{rem}} < 3\text{Msun} \Rightarrow \) neutron star
Neutron star mass limit: \( M_{\text{ns}} < 3 \text{ Msun} \)

\( M_i > 30 \text{ Msun} \Rightarrow M_{\text{rem}} > 3\text{Msun} \Rightarrow \) Black Hole (BH)

\textbf{Observations of remnants}

Planetary nebulae: \( \text{PN} \Rightarrow \text{emission lines}; \text{WD} \) in lower left of HRD

Neutron stars: \( \text{NS} \Rightarrow \text{pulsars} \) (Crab remnant from SN in 1054 AD)

Black holes: \( \text{BH accretion disk} \):
accretion of mass at rate $\dot{M}_{\text{acc}} \Rightarrow L_{\text{disk}} = \text{eff} \cdot \dot{M}_{\text{acc}} c^2$; 
\text{eff} = \frac{1}{4} \frac{R_{\text{bh}}}{R_{\text{acc}}} \approx 10\% \gg 0.7\% \text{ eff of H-burning}

typical $L_{\text{disk}} \sim 10^6 \text{ L}_{\odot}$; $T_{\text{disk}} \sim 10^6 \text{ K} \Rightarrow$ bright Xray source