20.1 Introduction

We have seen earlier that because massive MS stars have convective cores, the evolution across the HRD after H exhaustion in the core is relatively rapid. The core contraction ends when helium burning reactions begin under non-degenerate conditions. The star at this point is a red giant with He burning in a convective core and H burning in a shell outside the He core. The figure below shows the evolution in the HRD for population I stars with masses 3, 4, 5, 6, 9, 10 and 20 M$_\odot$ from the ZAMS to the end of core He burning.

Note the loops to the blue that occur in the tracks of the lower mass stars but are absent in the 20 M$_\odot$ track. The broken line indicates the approximate location of the Cepheid instability strip. δ Cephei stars
show regular oscillations in brightness on periods from 2 to 40 days. The period is well correlated with luminosity, which makes Cepheid variables very useful as distance indicators. The blue loops are very important for the existence of the Cepheid variables. For the low and high mass stars, the tracks cross the instability only once in the Hertzsprung gap. For the intermediate mass stars, the tracks cross the instability strip three times with the second and third crossings taking much longer than the first crossing. This can be seen in the figure below in which effective temperature is plotted against time for the $6 \, M_\odot$ model.

The horizontal broken lines are guides to show that the first crossing is much more rapid than the second and third crossings.

### 20.2 Composition changes in the core

Initially the He burning proceeds mainly through the $3\alpha$ reaction. As the $^{12}\text{C}$ abundance increases and the $^4\text{He}$ abundance diminishes, the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ becomes important. Hence the $^{12}\text{C}$ abundance at first increases in the core, reaches a maximum value, and then decreases again. This is shown in the figure below for the $6 \, M_\odot$ model.
20.3 Evolution after the end of core helium burning

Because of their convective cores, helium exhaustion in massive stars occurs over an extended region of the core. Hence the core contracts relatively rapidly, until either the electrons become degenerate or carbon burning begins. The critical initial mass that divides these two possibilities is sensitive to the details of convective mixing in the core. In the absence of convective overshooting, this mass is approximately $9 \, M_\odot$. For stars in which the electrons in the core become degenerate the further evolution is similar to that of lower mass stars. The star evolves through a helium shell burning phase followed by a sequence of thermal pulse cycles. As the core grows in mass, the radius and luminosity of the star both increase. This makes it easier for the star to lose mass, and the mass loss rate increases with time. The amount of mass lost determines the number of thermal pulses experienced on the TPAGB. The integrated amount of mass loss is constrained by the initial mass – final mass relation for white dwarfs in open clusters.
The complete evolutionary track of a population I 3 $M_\odot$ model is shown in the figure below.

A blowup of the TPAGB phase is shown below.

The figure below shows the stellar luminosity, the helium burning power and hydrogen burning power against age for the TPAGB phase.
We see that the interval between thermal pulses decreases with cycle number and also that the helium burning power increases with cycle number. These are both due to the increasing core mass.

The last complete thermal pulse cycle is shown below.

We see that the quiescent helium burning phase lasts for about 10% of the complete cycle. This is a consequence of hydrogen fusion producing ten times as much energy per unit mass as helium fusion.
Because of mass loss, the core does not get massive enough for carbon burning to occur and these stars end their lives as white dwarf stars with carbon oxygen cores.

20.4 Neutrino energy loss processes

There are a number of purely leptonic processes in which a pair of neutrinos can be emitted when an electron changes its momentum. The following processes can be important in stellar interiors.

20.4.1 Pair annihilation neutrino process \((e^+ + e^- \rightarrow \nu + \bar{\nu})\)

In very hot environments \((T > 10^9 \text{ K})\), electron-positron pairs can be created by photon processes. The electron-positron pairs are soon annihilated, usually giving two photons but once in about \(10^{19}\) cases a neutrino pair \((\nu \bar{\nu})\) is produced. If the plasma is not too dense, the neutrinos escape from the star without interaction. The loss rate is a complicated function but there are simple limiting cases for non-degenerate electrons:

\[
\begin{align*}
\epsilon_\nu &= 4.9 \times 10^{18} \frac{T_9^{3}}{\rho} \exp\left(-\frac{11.86}{T_9}\right) \quad T_9 < 1 \\
&= 4.6 \times 10^{15} \frac{T_9^9}{\rho} \quad T_9 > 3
\end{align*}
\]

Here the energy loss rate has units of \(\text{erg g}^{-1} \text{s}^{-1}\) and \(T_9 = T / 10^9 \text{ K}\).

Electron degeneracy reduces the neutrino loss rate by reducing the amount of phase space available for electron-positron pair production.

20.4.2 Plasma neutrino process \((\gamma_{\text{plasmon}} \rightarrow \nu + \bar{\nu})\)

A single photon cannot decay into a neutrino pair unless the neutrinos move in opposite directions. (This is because the photon has spin 1 and the neutrinos are spin \(\frac{1}{2}\) particles of opposite helicity.) If the neutrinos move in opposite directions, the decay cannot take place in vacuum because energy and momentum cannot be simultaneously conserved. The situation is changed in the presence of a stellar plasma. The plasma is a dielectric for photon propagation such that the dispersion relation between angular frequency and wave number is

\[
\omega^2 = k^2 c^2 + \omega_p^2,
\]

where \(\omega_p\) is the plasma frequency. In non-degenerate plasma, the plasma frequency is given by

\[
\omega_p = \frac{4\pi e^2 n_e}{m_e}.
\]

Electron degeneracy modifies the plasma frequency somewhat:
\[ \omega_p^2 = \frac{4\pi e^2 n_e}{m_e} \left[ 1 + \left( \frac{\hbar}{m_e c} \right)^2 \left( \frac{3\pi^2 n_e}{2} \right)^{2/3} \right]^{-1/2}. \] 

(20.4.4)

In non-degenerate stellar interiors the plasma frequency is so small in comparison to the thermal photon frequency, \( \hbar \omega_p = kT \), that photon processes are not significantly modified. In high density stellar interiors where the electrons are degenerate \( \omega_p \) can be comparable to \( \omega_{th} \).

The dispersion relation (20.4.2) is kinematically equivalent to the energy-momentum relation for a massive particle

\[ E^2 = p^2 c^2 + m^2 c^4. \]

Hence the electromagnetic wave acts like a relativistic Bose particle and when quantized is called a \textit{plasmon}. The plasmon has an equivalent rest mass

\[ m_{\text{plasmon}} = \frac{\hbar \omega_p}{c^2}. \]

(20.4.6)

For a given momentum, the electromagnetic wave in plasma has an excess energy \( \hbar \omega_p \) that allows photons to decay into neutrino pairs in which the neutrino and antineutrino move in opposite directions.

With the definitions

\[ x = \frac{\hbar \omega_p}{kT}, \]

(20.4.7)

and

\[ y = \frac{kT}{m_e c^2}, \]

(20.4.8)

the plasma neutrino loss rate has the limiting forms

\[ \rho \epsilon_{\nu} = 7.4 \times 10^{21} y^3 x^6 \quad x \ll 1 \]

\[ = 3.85 \times 10^{21} y^9 x^{15/2} e^{-x} \quad x \gg 1. \]

(20.4.9)

\( 20.4.3 \textbf{Photo-neutrino process} \ (\gamma + e \rightarrow e + \nu + \bar{\nu}) \)

This process is the analog of Compton scattering. The outgoing photon is replaced by a neutrino pair. This process competes with the pair annihilation process only at temperatures that are so low that electron-positron pairs are not created, and it competes with the plasma neutrino process only at densities so low that the plasmon rest mass is trivially small. Limiting forms for the energy loss rate are
\[ \rho \varepsilon_{\nu} = 0.98 \times 10^8 \frac{\rho}{\mu_e} T_{\nu}^8, \quad \text{nonrelativistic nondegenerate} \]
\[ = 4.8 \times 10^{11} \left( \frac{\rho}{\mu_e} \right)^{2/3} T_{\nu}^9, \quad \text{nonrelativistic degenerate} \] (20.4.10)

### 20.4.4 Bremsstrahlung neutrino process

This process is the analog of free-free emission. The outgoing photon is replaced by a neutrino pair. At high densities the energy loss rate is

\[ \varepsilon_{\nu} = 7.6 \times 10^8 \frac{Z^2}{A} T_{\nu}^6, \] (20.4.11)

where \( Z \) and \( A \) are the charge and mass number of the nucleus.

**Contours of log \( \varepsilon_{\nu} \)**

CO white dwarf core composition

![Contour plot](image-url)
The figure above is a contour plot of the neutrino loss rate per unit mass in units of erg g s\(^{-1}\). The broken lines show the run of temperature with density in the central regions of 10 and 50 M\(_{\odot}\) population I stars at the time of carbon ignition.

The figure above shows the dominant neutrino loss process in the log \(\rho\) – log \(T\) plane.

20.5 Evolution of stars more massive than 9 M\(_{\odot}\)

After the end of core helium burning, the CO core of the star contracts and heats. The density and temperature become large enough that neutrino losses become important. Neutrino losses affect the evolution in different ways depending on the degree of electron degeneracy. If the electrons are non-degenerate, then the energy loss from neutrino processes accelerates the contraction and heating. If the electrons are degenerate the neutrino energy loss results in cooling. Hence if carbon burning does occur, it does so under non-degenerate or only mildly degenerate conditions. For population I stars of mass of
mass 9 M_{\odot} and slightly higher, carbon ignites off center because the electrons at very center become degenerate and neutrino losses cool the inner regions.

We can use a simple model to estimate the critical core mass that separates the case in which contraction leads to increasing core temperature from the case in which the electrons in the core become degenerate, preventing further heating.

The equation of state is approximated by

\[ P = \frac{kT}{m_u} \left( \frac{\rho}{\mu_e} \right) + k_\gamma \left( \frac{\rho}{\mu_e} \right)^\gamma. \]  

(20.5.1)

Here we have ignored the ion pressure, which is a reasonable approximation since for advanced stages of evolution, \( \mu_{ion} \gg \mu_e \). The first term is the non-degenerate electron pressure that dominates at low densities. In the second term, \( \gamma \) and \( K_\gamma \) are not constants but vary to allow for both non-relativistic and relativistic degeneracy. At low density \( \gamma = 5/3 \), and decreases to 4/3 as the electrons become relativistic.

We assume that the core contracts homologously so that the central pressure and density are related by

\[ P_c = f G M_c \frac{2}{3} \rho_c^{4/3}, \]  

(20.5.2)

where \( f \) is a dimensionless constant of order unity. For uniform core density, \( f = (\pi/6)^{1/3} = 0.806 \), and for a polytrope of index \( n \),

\[ f = \frac{(4\pi)^{1/3}}{(n+1) \left( \frac{\xi}{\xi_1} \left[ \theta'(\xi) \right] \right)^{2/3}}. \]  

(20.5.3)

For \( n = 1.5 \) and \( n = 3 \), \( k = 0.478 \) and 0.364 respectively.

From equations (20.5.1) and (20.5.2), we find

\[ \frac{kT_c}{m_u} = f G M_c \frac{2}{3} \mu_e^{4/3} \left( \frac{\rho_c}{\mu_e} \right)^{1/3} - K_\gamma \left( \frac{\rho_c}{\mu_e} \right)^{\gamma - 1}. \]  

(20.5.4)

If the electrons are non-relativistically degenerate then \( \gamma - 1 = 2/3 \), and the central temperature has a maximum value, which can be found by setting the derivative of the right hand side to zero. However if the electrons are relativistically degenerate then \( \gamma - 1 = 1/3 \), and the central temperature will increase without limit provided the core mass is greater than a critical value which is essentially the Chandrasekhar mass.

The evolution of the core for various masses is shown in the diagram below. The thick line is the locus of the transition from non-degenerate to degenerate electrons, and the dashed line shows the threshold for carbon burning (taken to be where the nuclear energy generation rate exceeds the neutrino loss rate.
We see that if the core mass, $M_c$, is less than the Chandrasekhar mass, $M_{ch}$, the core temperature reaches a maximum then decreases. This is because the contraction of the core can be halted by electron degeneracy pressure. However if $M_c > M_{ch}$, the temperature increases without limit because electron degeneracy pressure is not sufficient to prevent the contraction. When we consider the nuclear energy production, we see that if $M_c$ is less than about $1 \, M_\odot$, carbon burning does not start and the core becomes a white dwarf. For larger core masses carbon burning begins under non-degenerate conditions.
So far we have considered the case in which the core mass is constant. If the envelope is sufficiently massive, helium shell burning will increase the mass of the core. Hence if $M_c$ is initially less than about 1 $M_\odot$, helium shell burning can increase $M_c$ to the point where carbon burning begins under degenerate conditions at a density of about $2 \times 10^9$ g cm$^{-3}$. This gives rise to a carbon core flash. The amount of energy released by carbon burning is approximately $10^{17}$ erg g$^{-1}$ (for a typical C to O ratio of 0.2 to 0.8 produced by the helium burning reactions). Degeneracy is lifted at a temperature of $1.4 \times 10^{10}$ K. To reach this temperature requires an energy release of $1.5 \times 10^{17}$ erg g$^{-1}$. Hence, unlike the helium core flash, the carbon is completely burned before degeneracy is lifted.

Of course, we cannot measure directly the mass of the core. From models we find the following results for the core mass at the end of helium burning in $Z = 0.02$ stars.

<table>
<thead>
<tr>
<th>Initial total mass ($M_\odot$)</th>
<th>Core mass at end of helium burning ($M_\odot$)</th>
<th>Core mass at beginning of carbon burning ($M_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.471</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.585</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.712</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.872</td>
<td>1.075</td>
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<td>8.5</td>
<td>1.081</td>
<td>1.237</td>
</tr>
<tr>
<td>9</td>
<td>1.174</td>
<td>1.342</td>
</tr>
<tr>
<td>10</td>
<td>4.175</td>
<td>4.300</td>
</tr>
</tbody>
</table>

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