STELLAR MAGNETOSPHERES

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Abstract. The term “magnetosphere” originated historically from early spacecraft measurements of plasma trapped by the magnetic field of earth and other planets. But over the years this concept has also been applied to the magnetically channeled wind outflows from magnetic stars. The review here describes the basic magnetohydrodynamics (MHD) approach used to model such stellar magnetospheres, with emphasis on the central competition between confinement by the magnetic field vs. expansion of the stellar wind outflow. A key result is that, for a star with a dipole surface field $B_\ast$, surface radius $R_\ast$, and asymptotic wind momentum $\dot{M}v_\infty$, this competition can be well characterized by a single “wind magnetic confinement parameter”, $\eta_\ast \equiv B_\ast^2 R_\ast^2 / \dot{M}v_\infty$. For large $\eta_\ast$, closed magnetic loops can confine parts of the wind up to an Alfvén radius $R_A \approx \eta_\ast^{-1/4} R_\ast$, leading to “magnetically confined wind shocks” that might produce the relatively hard X-ray emission seen in some magnetic stars. In rotating stars, $R_A$ also roughly characterizes the radius up to which material co-rotates with the underlying star. For the outflowing wind, the associated loss of angular momentum can lead to spindown in the stellar rotation over a time much shorter than the star’s evolutionary timescale. For confined material within $R_A$ but beyond the star’s Keplerian corotation radius $R_K$, the net centrifugal support against gravity can lead to a “rigidly rotating magnetosphere” composed of accumulating trapped wind. This can provide a natural explanation for the rotationally modulated Balmer line emission observed from magnetic Bp stars. Moreover, magnetic reconnection heating from episodic centrifugal breakout events might explain the occasional very hard X-ray flares seen from such stars. Overall, it seems clear that magnetic fields can play a strong role in confining and channeling such stellar wind outflows, providing a natural explanation for various observational signatures structure and variability in the winds and circumstellar envelopes of massive stars.
1 Introduction

The term “magnetosphere” was first coined in the context of the earth, to refer to the cavity that the earth’s magnetic field carves out in the interplanetary medium of plasma and magnetic field carried outward from the sun by the solar wind. It was a natural extension of the discovery, made by the first man-made satellites launched in the late 1950’s, that earth is enveloped by “radiation belts” (a.k.a. van Allen belts, after one of the principal discoverers James van Allen) consisting of ionized plasma and high-energy particles that are trapped and accelerated within earth’s magnetic field. Later interplanetary spacecraft detected similar (but much larger) magnetospheres around all the other strongly magnetized planets, namely the Jovian gas giants. And telescopic observations have since lead to extension of the general concept to the sun and other stars; for example, eclipse and coronagraph observations of the sun show quite vividly how the outward expansion of the solar corona into the solar wind is strongly influenced by the sun’s magnetic field, with closed magnetic loops of trapped plasma underlying radial streamers along field lines stretched open by the outward wind expansion (see figure 1). A principal focus of the review here will be on the magnetospheres that are inferred in other, more-massive stars of spectral type O and B, which have radiatively driven stellar winds that are much stronger than the gas-pressure-driven solar wind.

To set the context for this discussion of “stellar magnetospheres”, let us first contrast some of the key differences they have from planetary magnetospheres.

- **Outside-In Compression vs. Inside-Out Expansion.** The magnetospheres of the earth and other planets are compressed into a “tear-drop” shape by the external stress of the impacting solar wind. By contrast, in the sun and stars, the outward expansion of their winds exterts an inside-out stress that tends to stretch open their stellar magnetospheres, particularly in the outer regions, where the radial decline in strength makes the field too weak to confine the wind expansion.

- **Space Physics vs. Astrophysics.** Another difference is that while planetary magnetospheres fall generally within the discipline of “space physics”, stellar magnetospheres are studied from the perspective of “astrophysics”.

- **Local, in situ measurement vs. global, remote observations.** A key distinction in this regard is the main data constraining theoretical models. For the terrestrial or planetary magnetospheres, the constraints come from in situ measurements by orbiting or interplanetary spacecraft, whereas for stellar magnetospheres, they are mainly by remote telescopic observations. The former provide detailed, local information about the vector magnetic field as well as the full energy distribution functions for the electron and various ion components of the plasma. The latter provides more global constraints on the gas parameters (e.g. temperature and density), and in some cases (e.g. through polarization measurements) also for the magnetic field.
Fig. 1. White-light photograph of 1980 solar eclipse, showing how the million degree solar corona is structured by the solar magnetic field. The closed magnetic loops that trap gas in the inner corona become tapered into pointed “helmet” streamers by the outward expansion of the solar wind. Eclipse image courtesy Rhodes College, Memphis, Tennessee, and High Altitude Observatory (HAO), University Corporation for Atmospheric Research (UCAR), Boulder, Colorado. UCAR is sponsored by the National Science Foundation.

- **Plasma physics vs. (magneto)hydrodynamics models.** Driven by these detailed plasma data, theoretical models of earth’s magnetosphere thus focus on complex plasma physics processes aimed at understanding the electric field and current systems, and their role in transport and acceleration of the distinct electron and ion components of the plasma. By contrast, models of stellar magnetospheres are based on a more idealized single-fluid approach that focus on hydrodynamic or magnetohydrodynamic processes that set the global evolution of basic fluid properties like temperature, density, and velocity.

Note that the case of the sun involves both types of approach. Much of the modeling of the solar coronal expansion is based on various telescopic observations in a range of wavebands (e.g. white light coronagraph, X-ray and UV spectra) that show quite vividly the central role of magnetic fields in confining the hot coronal plasma in closed magnetic loops, and channeling the wind outflow in coronal “holes” of radial open field. But further away from the sun, this wind outflow can be measured *in situ* by interplanetary spacecraft at heliocentric distances ranging...
from about 0.3 AU (e.g. during the Helios mission) out to the boundaries of the heliosphere at ca. 100 AU (e.g. the Voyager missions). The solar case thus provides an important physical and conceptual framework for the study of stellar magnetospheres, for which there is no possibility of in situ plasma measurements, and for which even the telescopic observations are based on spatially unresolved spectra, albeit again often in a range of wavebands from radio to visible to X-ray.

Before discussing the specifics of current-day models of stellar magnetospheres, let us review the groundwork for the basic approximations involved in the magneto-hydrodynamic (MHD) approach used in such models.

2 The MHD model for Magnetized Plasmas

2.1 Basic Assumptions

As noted above, magnetized plasmas in astrophysics are commonly treated within the MagnetoHydroDynamics, or MHD, model. This involves several underlying assumptions and simplifications:

• Single fluid. The electrons and all the individual ion components of the plasma are so strongly coupled that together they act as a single fluid that globally is electrically neutral. A corollary is that the microscopic velocities of all components follow a Maxwell-Boltzmann distribution, characterized by a common single temperature and a common single bulk velocity for the overall fluid. In non-magnetized gases, such strong coupling is enforced by the frequent direct collisions among the individual atoms when the overall gas density is high. In ionized plasmas, the coupling involves coulomb interactions, which because of the long-range nature tends to be dominated by the cumulative effect of many weak individual interactions, or even coherent plasma waves that lead to “effective” collisions. The latter means that even at low densities for which the plasma might be considered “collisionless”, there can still be sufficient coupling to justify a single-fluid approach, at least as a first approximation.

• Large Scale in Length and Time. For both a neutral gas and an ionized plasma, an essential assumption of the above single-fluid treatment is thus that the model is being applied on length scales that are large compared to the effective collisional mean-free-path, and on time scales long compared to the effective collision time. But in the case of plasmas, there is also an assumption that the length scales are large compared other key scales, for example the “Debye length” (over which local charge of an ion is neutralized by the attraction of nearby electrons), or in the magnetized case the “ion gyro (a.k.a. cyclotron or Larmor) radius” (over which ions gyrate about the mean field). For typical conditions in stellar magnetospheres, these are both on human scales, e.g. meters, and thus very much smaller than characteristic astronomical lengths set by the associated stellar radius of millions of kilometers.
“Ideal” MHD with infinite conductivity. The free electrons of a plasma provide a natural carrier of electric current, leading thus to a very high conductivity. In “ideal MHD”, this conductivity is assumed to be effectively infinite. As discussed below, this leads to a frozen flux condition in which the plasma and field are effectively coupled together, except at very localized sites of magnetic reconnection between closely spaced fields of opposite polarity, where the ideal assumption of infinite conductivity breaks down.

2.2 Maxwell’s Equations and Ohm’s Law: Magnetic Induction vs. Diffusion

The equations of MHD are grounded in a reduced formulation of Maxwell’s equations governing the evolution of electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \),

\[
\nabla \cdot \mathbf{E} = 4\pi \rho_c \quad (2.1)
\]

\[
\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (2.2)
\]

\[
\nabla \cdot \mathbf{B} = 0 \quad (2.3)
\]

\[
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad (2.4)
\]

where the notation is standard, and following astronomical convention, we use CGS units. A basic MHD model assumes both no net macroscopic charge density \( \rho_c = 0 \), and negligible Maxwell displacement current \( (\partial \mathbf{E}/\partial t \to 0) \), since for a plasma bulk flow speed \( v \), this is of order \( (v/c)^2 \ll 1 \) compared to competing terms. For conductivity \( \sigma \), the current density \( \mathbf{J} \) in the stellar rest frame is then modeled in terms of a general Ohm’s law,

\[
\mathbf{J} = \sigma \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right). \quad (2.5)
\]

Applying this in eqn. (2.4) and taking the curl to apply that in eqn. (2.2), we find, after using some vector identities relating the curl, divergence, and gradient operators,

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} \right) + \frac{c^2}{4\pi} \nabla^2 \mathbf{B}. \quad (2.6)
\]

The first term on the right-hand-side represents the effect of magnetic induction, while the second term represents the effect of magnetic diffusion. The ratio between these can be characterized by a dimensionless, magnetic Reynolds number

\[
Re_m = \frac{\text{induction}}{\text{diffusion}} = \frac{4\pi \sigma L v}{c^2} \quad (2.7)
\]

where \( L \sim |\nabla|^{-1} \) represents a characteristic length for the gradient in the magnetic field, which for most of a volume of a stellar magnetosphere is of order the stellar radius, \( L \lesssim R \). The characteristic flow speed ranges from order the sound speed \( v \sim a \sim 20 \text{ km/s} \) to the the stellar escape speed \( v_{\text{esc}} \sim 600 \text{ km/s} \). Combined with the very high plasma conductivity \( \sigma \sim 10^{10} \text{ s}^{-1} \), this implies a huge magnetic Reynolds number, typically on order of \( Re_m \sim 10^{10} \gg 1 \).
2.3 Ideal MHD: Frozen Flux vs. Magnetic Reconnection

The above analysis shows that the magnetic diffusion is generally very small, of order $1/\text{Re}_m \ll 1$, compared to induction. A common approximation, known as *ideal MHD*, is effectively to neglect altogether the diffusivity, so that the evolution of magnetic field through most of the volume is solely through magnetic induction,

$$\frac{\partial \mathbf{B}}{\partial t} \approx \nabla \times (\mathbf{v} \times \mathbf{B}) .$$  \hspace{1cm} (2.8)

In the absence of any magnetic diffusion, eqn. (2.8) implies that the field and plasma are effectively tied to one another, a property that can be formally expressed through the so-called *Frozen Flux Theorem*. If we define the net magnetic flux through any surface $S$ via the areal integral,

$$\Phi_S = \int_S \mathbf{B} \cdot d\mathbf{A} ,$$  \hspace{1cm} (2.9)

then the frozen-flux theorem states that this does not change in time, i.e.

$$\frac{d\Phi_S}{dt} = 0 .$$  \hspace{1cm} (2.10)

The proof is a bit lengthy but straightforward, following from application of both the divergence and Stoke’s theorem from vector calculus, using both the induction eqn. (2.8) and the zero-divergence property of magnetic fields, eqn. (2.3).

The upshot of this frozen-flux theorem is that each local parcel of plasma is effectively forever locked to a single line of magnetic field. In the case that the plasma energy density is much greater than that of the magnetic field, any motion of the plasma can thus effectively drag the field lines around. But in the opposite limit that the field energy dominates over that of the plasma, then these field lines can act much like rigid pipes that guide, channel, and even confine the bulk flow of the plasma. Ultraviolet and X-ray images above solar limb provide vivid illustration of the complex field and the fine-structure of emitting material that threads localized field-lines.

Although this ideal MHD form for frozen-flux does generally apply over the broad volume of a stellar magnetosphere, it can break down in localized regions. For example, motions in the underlying dense plasma in the stellar atmosphere can force field lines together, leading to localized regions of strong gradient where the characteristic length scale is much smaller than in the overall volume. Through various complex, nonlinear plasma processes involving a combination of anomalous resistivity and runaway gradient steepening, the length scales can become small enough to make the local effective Reynolds number of order unity or less. This leads to a breakdown in the frozen flux condition that is characterized by *magnetic reconnection* between field lines. The associated dissipation of magnetic energy can represent an important heating source for the plasma. This reconnection can often be quite sudden, as in the case of *solar flares*, which result from the radiative emission of material that has been impulsively heated by the sudden release of magnetic energy through fast reconnection.
2.4 Lorentz Force and Equations of MHD

The dynamical effect of the field in guiding a plasma can be expressed quantitatively through the Lorentz force,

\[ f_B = \frac{J \times B}{c} = \left( \nabla \times \frac{B}{4\pi} \right) \times \frac{B}{4\pi} = \frac{B \cdot \nabla B}{8\pi} - \nabla \left( \frac{B^2}{8\pi} \right), \tag{2.11} \]

where the second equality follows from the Ampere-law Maxwell equation (2.4) in the absence of the Maxwell displacement current term. The last equality again uses vector identities to break this into two physically distinct terms, the first representing a kind of magnetic tension, and the latter a magnetic pressure.

This magnetic force then enters into the equation of motion for the bulk flow velocity \( \mathbf{v} \),

\[ \frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho} \nabla P + \frac{f_B}{\rho} + g_{\text{ext}}, \tag{2.12} \]

\( D/Dt = \partial/\partial t + \mathbf{v} \cdot \nabla \) represents the total change following along the flow; the density and velocity are also related via the equation of mass conservation,

\[ \frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} = 0. \tag{2.13} \]

The external acceleration \( g_{\text{ext}} \) represents the effect of additional body forces like gravity or radiative driving. The gas pressure obeys the ideal gas equation of state,

\[ P = \rho \frac{kT}{\mu} = \rho a^2 = (\gamma - 1) e, \tag{2.14} \]

with the second equality defining the isothermal sound speed \( a \) in terms of the temperature \( T \), molecular weight \( \mu \), and Boltzmann’s constant \( k \). The last equality introduces the internal energy density \( e \) in terms of the ratio of specific heats, which for a monotonic gas is just \( \gamma = 5/3 \); its evolution is described by an energy equation,

\[ \frac{De}{Dt} + \gamma e \nabla \cdot \mathbf{v} = \dot{Q}, \tag{2.15} \]

where \( \dot{Q} \) is the net plasma heating rate per unit volume.

Together with the auxiliary equation for zero-field divergence (2.3) and the magnetic induction equation (2.8), eqns. (2.11) – (2.15) represent the full set of governing equations for ideal MHD.

2.5 Alfvén Speed and Plasma Beta

In a non-magnetic gas or plasma, localized pressure perturbations propagate as a sound wave, with a speed proportional to the sound speed \( a \)\(^1\). In a magnetized

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\(^1\)Actually, \( a \) represents the propagation speed under the idealization that local heating and cooling (e.g. by radiation) keeps the perturbations isothermal. More commonly, in the absence of such external heating/cooling, the perturbations remain adiabatic, and propagate at a speed \( \sqrt{\gamma a} \), where the ratio of specific heats, \( \gamma = 5/3 \) for an ideal gas.
plasma, localized perturbations in the field can propagate in a new mode, known as an Alfvén wave, with a propagation at the Alfvén speed, defined by

$$V_A = \frac{B}{\sqrt{4\pi \rho}}. \quad (2.16)$$

Whereas sound waves are longitudinal and compressive, Alfvén wave are transverse and non-compressive, propagating along the field. And whereas the restoring force of sound waves is gas pressure, for Alfvén waves it stems from the magnetic tension of the field line; Alfvén waves are thus somewhat analogous to vibration waves on a taut string. The combination of magnetic tension with gas and magnetic pressure can also lead to two other hybrid waves, known as the fast and slow mode.

Note that the Alfvén speed can be related to the magnetic energy density and magnetic pressure,

$$E_{mag} = P_{mag} = \frac{B^2}{8\pi} = \frac{\rho V_A^2}{2}, \quad (2.17)$$

which is thus quite analogous to the relationship between sound speed and gas pressure, $P = \rho a^2$ (cf. eqn. (2.14).

For a static gas, a common way to relate the gas internal energy to magnetic energy is through the plasma “beta”,

$$\beta = \frac{P}{B^2/8\pi} = 2 \left(\frac{a}{V_A}\right)^2. \quad (2.18)$$

Note then that a low-beta plasma, with $\beta \ll 1$, is strongly magnetic, with $V_A \gg a$; whereas a high-beta plasma, with $\beta \gg 1$, is only weakly magnetic, with $V_A \ll a$.

In the context of the above concept of frozen flux, a low-beta plasma represents a case where the magnetic field effectively guides the plasma; a common example is the solar corona, which shows clear evidence of thin threads of dense plasma spread along local field lines. In contrast, in a high-beta plasma the gas can dominate and move the field; a common example in the solar photosphere, where convective motions sweep field lines to the border of convective cells, forming a complex magnetic network.

### 2.6 Wind Magnetic Confinement Parameter

As noted in the introduction, a general distinction between planetary and stellar magnetospheres is that the former tend to have an outside-in compression from the impacting solar wind, while the latter have an inside-out expansion from the acceleration of the wind from the stellar surface. For subsonic surface flows like convection, the competition between plasma and field is characterized by the ratio of thermal to magnetic energy, as represented by the plasma $\beta$ parameter.

But stellar wind outflows are generally quite supersonic, implying that the relevant plasma energy is in the form of an outflow kinetic energy, $\rho v^2/2$, rather than internal thermal energy, $e \sim P \sim \rho a^2$. Their ratio is proportional to the
square of the flow Mach number,
\[ \frac{\rho v^2}{2} \sim \frac{v^2}{a^2} \equiv M^2. \]  
(2.19)

Since stellar winds have \( M \gg 1 \), the competition between the outward wind acceleration and confinement of a closed loops of a surface field does not depend the plasma \( \beta \) that characterizes the internal vs. magnetic energy, but rather on the the ratio of the magnetic to kinetic energy densities (ud-Doula & Owocki, 2002; ud-Doula, 2003),
\[ \eta \equiv \frac{B^2}{8\pi \rho v^2 / 2} = \left( \frac{V_A}{v} \right)^2 \equiv \frac{1}{M_A^2}. \]  
(2.20)

The last two equalities emphasize that this energy ratio can also be cast as the square of the ratio of the Alfvén speed, \( V_A \equiv B/\sqrt{\lambda \rho} \), to flow speed, \( v \), i.e. as the inverse square of the Alfvénic Mach number, \( M_A \equiv v/V_A \). Note that this ratio is defined with the magnetic field in the numerator, so that, unlike the plasma \( \beta \), a higher value signifies a greater dynamical importance for the magnetic field.

In general, this field/wind energy ratio can have a complex spatial variation that depends on the geometry of the flow and field. But we can give a general description of its overall radial dependence by using the scalings for a steady, spherical wind with a mass loss rate,
\[ \dot{M} = 4\pi \rho vr^2, \]  
(2.21)

and parameterized velocity law,
\[ v(r) = v_\infty (1 - R_*/r)^b, \]  
(2.22)

where \( R_* \) is the stellar radius, \( v_\infty \) is the wind terminal speed, and the power index is typically \( b \approx 1 \). Let us further assume the magnetic field declines from its surface value \( B_* \) as a power-law in radius,
\[ B(r) = B_* \left( \frac{r}{R_*} \right)^{-q}, \]  
(2.23)

where, e.g., for a dipole field, \( q = 3 \). We can then write the radial variation in this field/wind energy ratio as
\[ \eta(r) = \eta_* \frac{(r/R_*)^{2-2q}}{(1 - R_*/r)^b}. \]  
(2.24)

where
\[ \eta_* \equiv \frac{B_*^2 R_*^2}{\dot{M} v_\infty} \]  
(2.25)

defines an overall Wind Magnetic Confinement Parameter. For \( \eta_* \ll 1 \), we can expect the radial wind outflow to effectively overwhelm the field, stretching it into
a nearly radial configuration everywhere. On the other hand, for $\eta_* \gg 1$, the field should dominate the outflow from the wind base.

In general the surface field strength will vary with colatitude, $B_\star(\theta)$; for example, in the simple dipole case the strength at the magnetic equator is only half that over the magnetic pole, $B_\star(90^\circ) = B_\star(0^\circ)/2$. The equatorial value is more relevant for wind confinement, since that is where the magnetic field is transverse to the radial outflow; but stellar surface fields are more commonly quoted in terms of a net radial flux that is more characteristic of a polar value. As such, a convenient scaling for evaluation of the equatorial confinement can be written,

$$\eta_* = 0.4 \frac{B_{100}^2 R_{12}^2}{M_6 v_8},$$

(2.26)

where the field strength is parameterized here in terms of its polar value, $B_{100} \equiv B_\star(0)/(100 \text{ G})$, with $M_6 \equiv \dot{M}/(10^{-6} \text{ M}_\odot/\text{yr})$, $R_{12} \equiv R_\star/(10^{12} \text{ cm})$, and $v_8 \equiv v_\infty/(10^8 \text{ cm/s})$ providing a convenient numerical evaluation based on characteristic scalings for an OB supergiant star (e.g. $\zeta$ Puppis). This indicates that significant magnetic confinement or channeling in such stars should require fields of order $\sim 100 \text{ G}$.

By contrast, in the case of the sun, the much weaker mass loss ($\sim 10^{-14} \text{ M}_\odot/\text{yr}$) means that even a much weaker global field ($B_\star \sim 1 \text{ G}$) is sufficient to yield $\eta_* \approx 50$, implying a substantial magnetic confinement of the solar coronal expansion. As discussed further in § 3, this is roughly consistent with the observed large extent of magnetic loops in optical, UV and X-ray images of the solar corona.

### 2.7 Alfvén Radius

A key point of the above analysis that the radial fall-off of the magnetic field energy is generally much steeper than for the wind energy. For example, for a dipole field with $q = 3$, we see that $B^2 \sim 1/r^6$, whereas for a wind near terminal speed, the energy decline is proportional to the density, which has only an inverse-square fall off with radius, $\rho \sim 1/r^2$. This means that even in the strong confinement case with $\eta_* \gg 1$, the wind outflow should always dominate the field at sufficiently large radii, $r \gg R_\star$.

To characterize this transition from magnetic to wind outflow dominance, it useful to define an Alfvén radius $R_A$ at which the field/wind energy ratio $\eta$ and the Alfvénic Mach number $M_A$ are both unity. Setting $\eta(R_A) \equiv 1$ in eqn. (2.24), we find for the canonical velocity-law index $b = 1$ that this Alfvén radius is given implicitly by

$$\left(\frac{R_A}{R_\star}\right)^{2q-2} - \left(\frac{R_A}{R_\star}\right)^{2q-3} = \eta_*.$$

(2.27)

For integer $2q$, this is just a simple polynomial, specifically a quadratic, cubic, or quartic for $q = 2, 2.5, \text{ or } 3$. But even for non-integer values of $2q$, the relevant solutions can be approximated (via numerical fitting) to within a few percent by
the simple general expression (ud-Doula et al., 2008),

\[
\frac{R_A}{R_*} \approx 1 + (\eta_* + 1/4)^{1/(2q-2)} - (1/4)^{1/(2q-2)}.
\]

(2.28)

For weak confinement, \(\eta_* \ll 1\), we find \(R_A \to R_*\), while for strong confinement, \(\eta_* \gg 1\), we obtain \(R_A \to \eta_*^{1/(2q-2)}R_*\). In particular, for the standard dipole case with \(q = 3\), we expect the strong-confinement scaling

\[
\frac{R_A}{R_*} \approx 0.3 + \eta_*^{1/4}; \quad \eta_* \gg 1, \; q = 3.
\]

(2.29)

Clearly \(R_A\) represents the radius at which the wind speed \(v\) exceeds the local Alfvén speed \(V_A\). But, as discussed further in §4, numerical MHD simulations (ud-Doula et al., 2008) for winds with a base dipole magnetic field show that it is generally just somewhat above (i.e., by 20-30\%) the maximum extent of closed loops in the magnetosphere, the radius for which follows a general scaling

\[
R_c \approx R_* + 0.7(R_A - R_*).
\]

(2.30)

### 2.8 Rotation Parameter and Kepler Co-Rotation Radius

For winds from rotating stars, the numerical simulations by ud-Doula et al. (2008) show that this closure radius \(R_c\) is also roughly the radius up to which the wind plasma is kept in a rigid-body rotation with the underlying star.

Let us thus next seek a similarly convenient parameterization for the stellar rotation. This can again be characterized in terms of a speed, namely the equatorial surface rotation speed \(V_{\text{rot}}\). But instead of relating that to the flow speed or Alfvén speed in the stellar wind, the stellar origin of rotation suggests it may be better to compare it to a speed representative of the gravity at the stellar surface. Specifically, let us thus define our dimensionless rotation parameter as

\[
W \equiv \frac{V_{\text{rot}}}{V_{\text{orb}}},
\]

(2.31)

where \(V_{\text{orb}} = \sqrt{GM/R_*}\) is the orbital speed near the equatorial surface. This characterizes the azimuthal speed needed for the outward centrifugal forces to balance the stellar surface gravity. It is only a factor \(1/\sqrt{2}\) less than the speed \(V_{\text{esc}}\) needed to fully escape the star’s surface gravitational potential.

For a non-magnetic rotating star, conservation of angular momentum in a wind outflow causes the azimuthal speed near the equator to decline outward as

\footnote{This is closely related to the commonly used rotation parameter \(\omega \equiv \Omega/\Omega_{\text{crit}}\), defined by the star’s angular rotating frequency \(\Omega\) relative to the value this would have as the star approaches “critical” rotation, \(\Omega_c\). Our choice here more directly relates to the additional local speed needed to propel material into Keplerian orbit, and avoids some subtle assumptions (e.g. rigid-body rotation using a Roche potential for gravity) about how the global stellar envelope structure adjusts to approaching the critical rotation limit.}
$v_\phi \sim 1/r$, meaning that rotation effects tend to be of diminishing importance in the outer wind.

By contrast, in a rotating star with a sufficiently strong magnetic field, magnetic torques on the wind can spin it up: for some region near the star, i.e., up to about the maximum loop closure radius $R_c$, they can even maintain a nearly rigid-body rotation, for which the azimuthal speed now increases outward in proportion to the radius,

$$v_\phi(r) = V_{\text{rot}} \frac{r}{R_\ast}; \quad r \lesssim R_c.$$  \hspace{1cm} (2.32)

As such, even for a star with surface rotation below the orbital speed, $W < 1$, maintaining rigid rotation will eventually lead to a balance between the outward centrifugal force from rotation and the inward force of gravity,

$$\frac{v_\phi^2(R_K)}{R_K} = \frac{GM}{R_K^2}.$$  \hspace{1cm} (2.33)

Combining this with eqns. (2.31) and (2.32) gives a simple expression for the associated “Kepler radius”,

$$R_K = W^{-2/3} R_\ast.$$  \hspace{1cm} (2.34)

Unsupported material at radii $r < R_K$ will tend to fall back toward the star, but any material maintained in rigid-rotation to radii $r > R_K$ will have a centrifugal force that exceeds gravity, and so will tend to be propelled further outward. Indeed, any corotating material above an “escape radius”, which is only slightly beyond the Kepler radius,

$$R_E = 2^{1/3} R_K,$$  \hspace{1cm} (2.35)

will have sufficient rotational energy to escape altogether the local gravitational potential, unless, of course, temporarily held down by the magnetic field. § 4 summarizes recent MHD models of the magnetospheres of hot, massive stars, with emphasis on the combined effects of rotation and magnetic confinement.

### 2.9 Angular Momentum Loss and Stellar Rotation Spindown

The addition of a magnetic fields to a rotating stellar wind outflow can substantially increase the angular momentum carried out by the wind, and thus lead to a much more rapid spindown of the stellar rotation. A pioneering analysis of this angular momentum loss for the solar wind was carried out by Weber & Davis (1967, hereafter WD67). They used a simple radial model of the magnetic field emanating from the rotating solar surface to study the resulting angular momentum in the equatorial plane of the pressure-driven solar wind. As shown in the left panel of figure 2, the field forces the azimuthal wind speed to initially increase somewhat from the $v_\phi \approx 2$ km/s near the solar surface at $r = R_\odot$. This is the result in the increased momentum arm from the magnetic field, and leads to enhancement in the angular momentum loss of the wind. But, as illustrated in the right panel of figure 2, a major surprise is that the majority of this increased angular momentum is not associated with the azimuthal motion of the wind material
Fig. 2. Left: Radial variation of azimuthal velocity of solar wind based on 1D equatorial plane, radial-field analysis by WD67. Right: The corresponding radial variation angular momentum-per-unit mass carried by the magnetic field Poynting stress (dashed) and by the wind material itself (full).

itself, but rather the Poynting stresses associated with the azimuthal distortion of the magnetic field!

A further key result of this WD67 analysis is that the total angular momentum loss is given by

\[ \dot{J} = \dot{M} \Omega R_A^2, \]

where \( \dot{M} \) is the mass loss rate, \( \Omega \) is the star’s angular rotation frequency, and \( R_A \) is the Alfvén radius at which the wind flow speed becomes equal to the Alfvén speed, \( v(R_A) = V_A \). Note that this is just the angular momentum loss that would obtain from just the fluid if the field were to enforce rigid-body rotation out to the Alfvén radius, and then shut off abruptly to allow angular momentum conservation for all radii beyond,

\[ v_\phi = \Omega r \quad ; \quad R_\odot \leq r \leq R_A \]
\[ = \Omega \frac{R_A^2}{r} \quad ; \quad r \geq R_A. \]

However, it should be emphasized that, in this WD67 analysis for a simple monopole, radial field, rigid rotation is not actually maintained to \( R_A \), and 80% of the angular momentum loss is actually through the magnetic field, not the plasma itself.

On the other hand, in MHD models of wind confinement by a dipole magnetic field, material trapped in closed loops does tend to corotate with the star out to about the associated Alfvén radius, which generally just somewhat (ca. 20%) above the maximum loop “closure” radius \( R_c \). Moreover, such MHD models suggest that the total angular momentum loss does follow the general scaling given by eqn. (2.36), corrected by a factor of order unity to account for latitudinal variations and other effects. For a star with moment of inertia \( I = f M_* R_*^2 \), where the stellar structure factor \( f \approx 0.1 \), this leads to a convenient simple scaling for the
characteristic spin down time for the star’s rotational angular momentum $J = I \Omega$,

$$
\tau_J \equiv \frac{J}{\dot{J}} \approx \frac{I}{MR_A^2} \approx \frac{f}{\sqrt{\eta_*}} \tau_M .
$$

(2.38)

The last equality uses a simplified form for the dipole scaling for the Alfvén radius of eqn. (2.29), $(R_A/R_*)^2 \sim \sqrt{\eta_*}$, to write this spin down time in terms of the stellar mass loss timescale $\tau_M$. For magnetic Bp stars with $\tau_M \approx 10^{10}$ years but $\eta_* > 10^6$, the spin down time can be of order a million years or less.

On the other hand, in the WD67 monopole field model for the solar wind, the Alfvén radius $R_A \approx 20 R_\odot$. For the solar wind mass loss timescale $\tau_M \approx 10^{14}$ years, this leads to a spin-down time that is comparable to the age of the sun, $\tau_J \approx 2.5 \times 10^9$ years. This is thought to provide the basic explanation for why the sun is such a relatively slow rotator.

3 Coronal Expansion and Solar Wind

The sun’s energy is generated by hydrogen fusion in the hot, ca. $10^7$ K, solar core, but as this energy diffuses outward the temperature steadily declines, reaching about 5700 K near the visible surface, or photosphere. Careful spectroscopic observations in various wavebands from the X-ray to radio show, however, that in the upper layers above this visible photosphere, the temperature again increases, indeed jumping rather abruptly back up to temperatures of more than $10^6$ K in the rarefied solar corona. The high gas pressure associated with this very high temperature makes the solar corona tend to expand outward against the inward pull of gravity, ultimately transitioning into a supersonic solar wind that can be measured in situ by interplanetary spacecraft. Before considering the role of magnetic fields in structuring and channeling the corona and solar wind, let us review the basic physics underlying this coronal expansion.

3.1 Pressure Extension of Spherical, Hydrostatic Corona

In the solar atmosphere, the stratification of gas pressure $P$ with radius $r$ establishes a hydrostatic equilibrium that supports the local mass density $\rho$ against gravity $g$. In a simple isothermal case, both pressure and density decline exponentially with height $z$,

$$
\frac{P(z)}{P_*} = \frac{\rho(z)}{\rho_*} = e^{-z/H_*},
$$

(3.1)

where the characteristic scale height,

$$
H_* = \frac{P}{|dP/dr|} = \frac{a^2}{g} = \frac{2a^2}{v_{esc}^2} R_\odot .
$$

(3.2)

The last equality casts this in terms of the solar radius $R_\odot$ times a fraction that depends on the squared ratio of the sound speed $a$ to escape speed $v_{esc}$. For example, the solar photosphere with temperature $T \approx 6000$ K has a sound speed
\[ a \approx 9 \text{ km/s} \] that is much smaller than the surface escape speed \( v_{\text{esc}} \approx 600 \text{ km/s} \). This gives a pressure scale height of \( H \approx 300 \text{ km} \), which is less than 1/1000 of the solar radius, \( R_\odot \approx 700,000 \text{ km} \). (This is the basic reason the visible solar photosphere has such a sharp edge.)

In contrast, for the multi-million-degree temperature of the solar corona, this scale height becomes more comparable to the solar radius; for example, for the typical solar coronal temperature of 2 MK, the ratio is about 1/7. In considering a possible hydrostatic stratification for the solar corona, it is thus now important to take explicit account of the radial decline in gravity,

\[
\frac{d \ln P}{dr} = -\frac{GM_*}{a^2 r^2}.
\] (3.3)

Because of the much broader spatial scales involved in this case, let us consider a slightly more general model for which the temperature has a power-law radial decline, \( T/T_* = a^2/a_*^2 = (r/R_*)^{-q} \). Integration of eqn. (3.3) then yields

\[
\frac{P(r)}{P_*} = \exp \left( \frac{R_*}{H_* (1-q)} \left( \frac{R_*}{r} \right)^{1-q} - 1 \right) .
\] (3.4)

A key difference from the exponential stratification of a nearly planar photosphere (cf. eqn. 3.1) is that the pressure now approaches a finite value at large radii \( r \to \infty \),

\[
\frac{P_\infty}{P_*} = e^{-R_*/H_* (1-q)} = e^{-14/T_6 (1-q)} ,
\] (3.5)

where the latter equality applies for solar parameters, with \( T_6 \) the coronal base temperature in units of \( 10^6 \text{ K} \). This gives \( \log(P_\infty/P_* \approx 6/T_6/(1-q)) \).

To place this in context, we note that a combination of observational diagnostics give \( \log(P_{TR}/P_{ISM}) \approx 12 \) for the ratio between the pressure in the transition region base of the solar corona and that in the interstellar medium. This implies that a hydrostatic corona could only be contained by the interstellar medium if \( (1-q)T_6 < 0.5 \). Specifically, for the conduction-dominated temperature index \( q = 2/7 \), we require \( T_6 < 0.7 \). Since this is well below the observational range \( T_6 \approx 1.5 - 3 \), the implication is that a conduction-dominated corona cannot remain hydrostatic, but must undergo a continuous expansion, known of course as the solar wind.

### 3.2 Isothermal Model for Solar Wind

Since a hydrostatic corona is not tenable, we must replace the hydrostatic equilibrium with an equation of motion for acceleration of a solar wind, assuming for simplicity an isothermal, steady-state, spherical outflow,

\[
\left( 1 - \frac{a^2}{v^2} \right) \frac{dv}{dr} = \frac{2a^2}{r} - \frac{GM_*}{r^2} ,
\] (3.6)
Fig. 3. Solution topology for an isothermal coronal wind, plotted via contours of the integral solution (3.8) with various integration constants $C$, as a function of the ratio of flow speed to sound speed $v/a$, and the radius over critical radius $r/r_c$. The heavy curve drawn for the contour with $C = -3$ represents the transonic solar wind solution.

Note that this uses the ideal gas law for the pressure $P = \rho a^2$ to eliminate the density through the steady-state of an unspecified, but constant overall mass loss rate $\dot{M} \equiv 4\pi \rho vr^2$.

The right-hand-side of eqn. (3.6) has a zero at the critical radius,

$$\frac{r_c}{R_\odot} = \frac{GM_\odot}{2a^2R_\odot} = \frac{v_{esc}^2}{4a^2} = \frac{7}{T_6}.$$  \hspace{1cm} (3.7)

At this radius, the left-hand-side of eqn. (3.6) must likewise vanish, either through a zero velocity gradient $dv/dr = 0$, or through a sonic flow speed $v = a$. Direct integration of eqn. (3.6) yields the general solution

$$F(r, v) \equiv \frac{v^2}{a^2} - \ln \frac{v^2}{a^2} - 4\ln \frac{r}{r_c} - \frac{4r}{r_c} = C,$$ \hspace{1cm} (3.8)

where $C$ is an integration constant. Using a simple contour plot of $F(r, v)$ in the velocity-radius plane, figure 3 illustrates the full “solution topology” for an isothermal wind. Note that for $C = -3$, two contours cross at the critical radius.
\((r = r_c)\) with a sonic flow speed \((v = a)\). The positive slope of these represents the standard solar wind solution, which is the only one that takes a subsonic flow near the surface into a supersonic flow at large radii.

### 3.3 Dependence of Solar Wind Mass Loss Rate on Energy Addition

Note that, since the density has scaled out of the controlling equation of motion (3.6), the wind mass loss rate \(\dot{M} \equiv 4\pi \rho vr^2\) does not appear in this isothermal wind solution. An implicit assumption hidden in such an isothermal analysis is that, no matter how large the mass loss rate, there is some source of heating that counters the tendency for the wind to cool with expansion. As we now discuss, determining the overall mass loss rate requires a model that specifies the location and overall level of this heating.

For a purely thermally driven wind, the total energy change from a base radius \(R_\odot\) to a given radius \(r\) depends on the integral of the net volume heating \(\dot{Q}_{\text{net}}\),

\[
\dot{M} \left[ \frac{v^2}{2} - \frac{v^2_{\text{esc}}}{2} \frac{R_\odot}{r} + \frac{5a^2}{3} \right]_{R_\odot}^r = 4\pi \int_{R_\odot}^r r^2 \dot{Q}_{\text{net}} dr' + 4\pi \left[ R_\odot^2 F_{\odot} - r^2 F_c \right],
\] (3.9)

where \(F_c\) is the conductive heat flux density. Note that without the energy source terms on the right-hand-side, the square bracket term on the left-hand-side would be constant, representing a case in which adiabatic cooling would not sustain a pressure-driven expansion. The expression here of the gravitational and internal enthalpy in terms the associated escape and sound speed \(v_{\text{esc}}\) and \(a\) allows convenient comparison of the relative magnitudes of these with the kinetic energy term, \(v^2/2\). For a typical coronal temperature of a few MK, \(a^2 < v^2_{\text{esc}}\), implying that the gravitational term dominates in the subsonic, nearly static base (where \(v \approx 0\)). Far from the star, this gravitational term vanishes, and so for a supersonic solar wind the kinetic energy term dominates. For a thermally driven wind, we can thus effectively ignore the enthalpy term in the global wind energy balance,

\[
\dot{M} \left( \frac{v^2_{\infty}}{2} + \frac{v^2_{\text{esc}}}{2} \right) \approx 4\pi \int_{R_\odot}^r r^2 \dot{Q}_{\text{net}} dr' + 4\pi R_\odot^2 F_{\odot} \equiv E_{h,\infty}.
\] (3.10)

Since typically \(v_{\infty} \approx v_{\text{esc}}\), we see that the mass loss rate is roughly set by the total net energy addition,

\[
\dot{M} \approx \frac{E_{h,\infty}}{v_{\text{esc}}^2}.
\] (3.11)

### 3.4 Extended Energy Addition and High-Speed Wind Streams

More quantitative analyses solve for the wind mass loss rate and velocity law in terms of some model for both the level and spatial distribution of energy addition into the corona and solar wind. The specific physical mechanisms for the heating are still a matter of investigation, but one quite crucial question regards the relative fraction of the total base energy flux deposited in the subsonic vs. supersonic
portion of the wind. Models with an explicit energy balance generally confirm a close link between mass loss rate and energy addition to the subsonic base of coronal wind expansion. By contrast, in the supersonic region this mass flux is essentially fixed, and so any added energy there tends instead to increase the energy-per-mass, as reflected in asymptotic flow speed $v_\infty$.

An important early class of solar wind models assumed some localized deposition of energy very near the coronal base, with conduction then spreading that energy both downward into the underlying atmosphere and upward into the extended corona. Overall, such conduction models of solar wind energy transport were quite successful in reproducing interplanetary measurements of the speed and mass flux of the “quiet”, low-speed ($v_\infty \approx 350 - 400$ km/s) solar wind.

However, such models generally fail to explain the high-speed ($v_\infty \approx 700$ km/s) wind streams that are thought to emanate from solar “corona holes”. Such coronal holes are regions where the solar magnetic field has an open configuration that, in constrast to the closed, nearly static coronal “loops”, allows outward, radial expansion of the coronal gas. To explain the high speed streams, it seems that some substantial fraction of the mechanical energy propagating upward through coronal holes must not be dissipated as heat near the coronal base, but instead must reach upward into the supersonic wind, where it provides either a direct acceleration (e.g. via a wave pressure that gives a net outward $g_x$) or heating ($\dot{Q}_h > 0$) that powers extended gas pressure acceleration to high speed.

Observations of such coronal hole regions from the SOHO satellite (Kohl et al., 1999; Cranmer et al., 1999) show temperatures of $T_p \approx 4 - 5$ MK for the protons, and perhaps as high as 100 MK for minor ion species like oxygen. The fact that such proton/ion temperatures are much higher than the ca. 2 MK inferred for electrons shows clearly that electron heat conduction does not play much role in extending the effect of coronal heating outward in such regions. The fundamental reasons for the differing temperature components are a topic of much current research; one promising model invokes ion-cyclotron-resonance damping of magnetohydrodynamics waves (Cranmer, 2000).

In general, it seems clear that magnetic fields play a key role in the storage, transport, channelling, and dissipation of mechanical energy for coronal heating. Monitoring by orbiting coronagraphs show the corona to be highly structured and variable on a range of spatial and temporal scales, with a constant jostling of field guided loops, punctuated by sporadic flares and/or coronal mass ejection events associated with release of energy through magnetic reconnection. In situ measurements by interplanetary spacecraft show the resulting solar wind is likewise highly variable, sometimes as a result of temporal changes induced in the coronal source regions, and sometimes in the form of “corotating interaction regions” between slow- and high-speed solar wind streams emanating from different spatial region of the rotating solar surface.
3.5 Magnetic Confinement of Solar Wind

The magnetic confinement parameter defined in § 2.6 actually provides a quite convenient way to quantify this role of magnetic fields in structuring the solar coronal expansion. The sun’s surface magnetic field is spatially very complex, but on the large scale of coronal loops, the net mean surface strength is about $B_s \approx 1$ G. Moreover, in situ measurements by interplanetary spacecraft give a typical solar flow speed $v \approx 400$ km/s and proton number density $n_p = 3 \text{ cm}^{-3}$ at the 1 AU distance earth’s orbit. Assuming comparable values over a full sphere of this radius, this gives a solar wind mass loss rate of roughly $\dot{M}_\odot \approx 10^{-14} M_\odot/\text{yr}$. Application of these parameters in eqn. (2.26) implies a magnetic confinement of $\eta_\odot \approx 50$.

For a dipole field with $q = 3$, this would imply an Alfvén radius

$$\frac{R_A}{R_\odot} \sim \eta_\star^{1/(2q-2)} \sim 50^{3/4} \sim 2.6 ; \quad q = 3 \text{ (dipole)},$$

which seems higher than the typical heights of closed loops seen in the eclipse image in figure 1. Note however that the magnetic field structure in this image is clearly much more complex than a simple dipole, implying $q > 3$, and thus giving

$$\frac{R_A}{R_\odot} \sim 50^{1/6} \sim 1.9 ; \quad q = 4 \text{ (quadrupole)}$$
$$\sim 50^{1/8} \sim 1.6 ; \quad q = 5 \text{ (octupole)},$$

both of which seem more consistent with the typical height of closed coronal loops.

Note that, for a typical coronal temperature $T_\star \approx 2$, these loop heights are all well below the expected transonic, critical radius $r_c \approx 3.5 R_\odot$ given by eqn. (3.7). The upshot is thus that the coronal gas in closed magnetic loops can remain in a static hydrostatic equilibrium. As discussed further in § 4, this is a major difference form the situation in radiatively driven winds, which are quickly accelerated to supersonic upflows that the field then channels into strong shock collisions at the tops of closed loops.

3.6 MHD Models of Coronal Wind Expansion

Efforts to develop MHD models of the coronal expansion date back to the pioneering work of Pneuman & Kopp (1971). Long before the advent of modern MHD simulation codes, developed an iterative algorithm for finding a self-consistent dynamical solutions for coronal wind expansion in the presence of a large-scale surface field, taken for simplicity to be a dipole. The left panel of figure 4 shows how the initial dipole surface field (dashed lines) is stretched open by the solar coronal expansion, forming the characteristic pointed helmet streamer at the tops of closed loops. The proximity of opposite north/south magnetic polarity in the outflowing wind along the magnetic equator induces a current sheet that extends out into the interplanetary flow of the solar wind.
Fig. 4. MHD models of the coronal expansion. Left: the original iteration model of Pneuman & Kopp (1971) that derived the dynamical MHD field (solid lines) from initial dipole configuration (dashed lines). Note how the solar wind expansion has stretched open the tops closed magnetic loops into a point “helmet” stream configuration reminiscent of eclipse photos, with the opposite north/south field polarity separation by a current sheet. Middle: Modern 3D MHD simulation by Mikić et al. (2007) based on observationally inferred photospheric field just prior to the 29-Mar-2006 solar eclipse. Right: White-light eclipse image taken by High Altitude Observatory and Rhodes College eclipse team. Note the good overall correspondence between the observed white configuration of bright coronal streamers and dark coronal holes with the computed regions of closed loops and radially open magnetic field.

The middle panel provides a nice example of what is now possible with modern MHD codes running on state-of-the-art computer clusters. This shows a full 3D MHD simulation by Mikić et al. (2007) that uses the actual observational inferred magnetic structure from the solar photosphere to provide the lower boundary condition to a full dynamical simulation of the coronal expansion. Indeed, by using the inferred photospheric field for the few days before the 29-Mar-2006 solar eclipse, the simulation effectively predicted the coronal magnetic field structure during the eclipse. Comparison with the right panel showing the actual white-light eclipse image shows the quite impressive overall agreement between the simulation prediction and the observed corona. The regions of closed magnetic cloops correspond quite closely with the observed bright coronal streamers, while the regions of radially open magnetic field correspond to the relatively dark coronal holes. Despite this success, it should be noted that the MHD model did use a relatively simple, parameterized form for the coronal temperature. Ideally, future work should account also for the basic mechanism for the coronal heating along with the dynamical channeling effect of the magnetic field.

Moreover, it should again be emphasized that, while the overall geometry of the coronal expansion during slowly evolving epochs can be modeled in terms of quasi-steady MHD simulations like those above, there are times when the corona exhibits dramatic changes, marked by strong flares and “coronal mass ejections”. These are signatures of fast magnetic reconnection that can occur following the
build-up of magnetic stresses associated with the convective motions at the footpoints of coronal field. Much like magnetotail shedding and other disruptions in the magnetospheres of earth and other planets, such events demonstrate that the interaction between magnetic field and plasma flow is generally a variable, dynamical process.

4 Magnetospheres of Massive-Stars

4.1 Background

Massive, luminous, hot stars have strong, radiatively driven stellar winds (Lucy and Solomon, 1970; Castor et al., 1975), with flow speeds ranging up to one percent of the speed of light, and mass loss rates ranging up to a billion times that of the solar wind. Their high surface temperatures mean that such stars lack the hydrogen recombination convection zone that induces the magnetic dynamo cycle of cooler, solar type stars (e.g., Parker, 1955). Hot stars have thus been classically treated as having a hydrostatic radiative envelope that is nonmagnetic and spherically symmetric, suggesting that their dynamical, radiatively driven stellar wind should likewise be nonmagnetic, spherically symmetric and steady-state.

But in fact spectroscopic monitoring of lines formed in such hot-star winds show them to be generally quite variable, commonly exhibiting discrete absorption components that typically recur on times scales comparable to the stellar rotation period. Such wind modulation could well be the result of a weak to moderate magnetic field at the stellar surface, which induces faster and slower wind streams that the stellar rotation causes to collide in spiral Corotating Interaction Regions (“CIRs”; Mullan, 1984; Cranmer and Owocki, 1996; Owocki & ud-Doula, 2004), much as is observed in situ for the solar wind (Hundhausen, 1973).

Indeed, over the years spectropolarimetric measurements have led to direct detection of quite strong fields in some hot stars. A first example was the detection (Landstreet and Borra, 1978) of a strong (∼ 10 kG), oblique-dipole magnetic field in the helium-strong B2p star σ Ori E. Subsequent observations of other members of this helium-strong class (so named on account of their elevated photospheric He abundances) revealed similar-strength fields in several additional objects (Borra and Landstreet, 1979). In recent years, advances in spectropolarimetric techniques have led to the discovery of weaker fields in other early-type systems, including Be emission-line stars (e.g., ω Ori – Neiner et al., 2003a), slowly-pulsating B stars (e.g., ζ Cas – Neiner et al., 2003b), and the more massive O-type stars (e.g., θ1 Ori C – Donati et al., 2002). In conjunction with the indirect evidence from wind-line variability, it now seems plausible that most or even all hot stars might harbor magnetic fields, albeit at levels that fall below historical and present-day detection thresholds.

In most cases these polarimetry observations are well fit by a simple dipole surface field with an axis that is tilted by some fixed angle β relative to the stellar rotation axis. The dipole nature and relative constancy of the inferred field amplitude and orientation – in some cases extending now over three decades –
seems clearly to preclude the kind of active convective dynamo that gives rise the magnetic activity cycles in solar type stars. As such, the source of hot-star fields remains uncertain, with suggested possibilities ranging from a fossil origin (as in Mestel, 2003), to slow buoyant diffusion to the surface of fields generated in the star’s convective core (as in Charbonneau and MacGregor, 2001; MacGregor and Cassinelli, 2003).

This remainder of this section reviews recent efforts to develop dynamical models for the effects of such a surface dipole field on the radiatively driven mass outflow.

### 4.2 MHD Simulations of Wind Outflows from Magnetic Hot Stars

Recent efforts have applied the zeus-3D MHD code (for details of the code, see Stone & Norman, 1992) to 2D axisymmetric simulations of the dynamical interplay between a dipole stellar magnetic field and a radiatively driven, hot-star wind (ud-Doula & Owocki, 2002; ud-Doula, 2003; Owocki & ud-Doula, 2004; ud-Doula et al., 2008). The results generally confirm that the overall effectiveness of magnetic fields in channeling a stellar wind outflow can be characterized by the wind magnetic confinement parameter $\eta^* \equiv B^2 R^2 / (M v_\infty)$ defined by scaling analysis in §2.6.

Initial MHD simulations (ud-Doula & Owocki, 2002) assumed, for simplicity, that radiative heating and cooling would keep the wind outflow nearly isothermal at roughly the stellar effective temperature. But to model the X-ray emission from shocks that form from the magnetic channeling and confinement, subsequent efforts (ud-Doula, 2003; Gagné et al., 2005) have relaxed this simplification to include a detailed energy equation that follows the radiative cooling of shock-heated material. In particular, building upon the initial suggestion by Babel & Montmerle (1997) that such Magnetically Confined Wind Shocks (MCWS) might explain the relatively hard X-ray spectrum observed by Rosat for the O7V star $\theta^1$ Ori C, (Gagné et al., 2005) have applied such energy-equation, MHD simulations toward a detailed, dynamical model of the more extensive Chandra X-ray observations of this star. Based on the recent spectropolarimetric measurement (Donati et al., 2002) of a ca. 1100 G field for $\theta^1$ Ori C, combined with empirical and theoretical estimates of the wind momentum and stellar radius, the simulations assume a moderately large magnetic confinement parameter, $\eta^* \approx 10$.

Fig. 5 illustrates results at two time snapshots, representing respectively a relatively early, simple phase ($t=80$ ks; left two panels), and a typical later, more complex phase ($t=180$ ks; right two panels). In each figure, the greyscales represent the spatial distribution of log density (first and third panel) and log temperature (second and fourth panel), with superposed lines representing the magnetic field. After introduction of the field, the left panels show the initial wind response is to stretch open the field lines in the outer region, but to be channeled toward the magnetic equator by the closed loops in the inner region. Within these closed loops, the flow from opposite hemispheres collides to make strong, X-ray emitting shocks, yielding a nearly symmetric structure that, at this time snapshot, is quite similar to what was predicted in the semi-analytic, fixed-field models of Babel & Montmerle
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**Fig. 5.** MHD simulations of the MCWS model for \( \theta^1 \) Ori C, showing the logarithmic density \( \rho \) and temperature \( T \) in a meridional plane. Left: at a time 80 ksecs after the initial condition, the magnetic field has channeled wind material into a compressed, hot disk at the magnetic equator. Right: at a time 180 ksecs, the cooled equatorial material is falling back toward the star along field lines, in a complex ‘snake’ pattern. The darkest areas of the temperature plots represent gas at \( T \sim 10^7 \) K, hot enough to produce relatively hard X-ray emission of a few keV.

(1997). However, such a simple, symmetric compression is only transient, lasting only a few 10 ksec, after which it evolves to the much more complex structure shown in the right panels of fig. 5. This is because once shocked material at the tops of loops cools, its support against gravity by the magnetic tension along the convex field lines is inherently unstable, leading to a complex pattern of fall back along the loop lines down to the star. However, when averaged over time (which here might roughly substitute for averaging over azimuth in a more realistic 3-D simulation), the overall level of X-ray emission turns out to be quite similar to what’s obtained from the simple, symmetric state represented by the left panels of Fig. 5.

Overall, the associated X-ray emission of this MHD model matches quite well the key properties of the *Chandra* observations for \( \theta^1 \) Ori C (Gagné et al., 2005), including: the relatively hard X-ray spectrum that arises from the high post-shock temperatures \( T \sim 20 – 30 \) MK; the relative lack of broadening or blue-shift from X-ray lines emitted from the nearly static, shock-heated material; and the X-ray light curve eclipse that stems from the moderate source radius \( r \approx 1.5R_* \) for the bulk of the X-ray emission.

### 4.3 Wind Spin-Up from Dipole Aligned with Stellar Rotation Axis

Let us next examine the nature of magnetic channeling for the winds from rotating hot stars. One particular issue is whether a large-scale magnetic field could spin-up the stellar wind outflow into a “Magnetically Torqued Disk” (MTD), as advocated by (Cassinelli et al., 2002).

As noted above, MHD simulations (ud-Doula & Owocki, 2002; ud-Doula, 2003) indicate that a dipole magnetic field can confine the flow within closed loops that extend out to just below the Alfvén radius, \( R_A \approx \eta_1^{1/4}R_* \). In rotating models such closed loops tend also to keep the outflow in rigid-body rotation with the under-
lying star, and so \( R_A \) also roughly represents the radius of maximum rotational spin-up of the wind azimuthal speed. The scalings derived in §2.8 suggest then that a likely necessary condition for propelling outflowing material into a Keplerian disk is to choose a combination of parameters for magnetic confinement vs. stellar rotation such that \( R_K < R_A < R_E \). As a sample test case, let us focus here on the specific combination \( \eta_* = 10 \) and \( W = 1/2 \), which gives the sequence \( \{ R_K, R_A, R_E \} = \{ 1.59, 1.78, 2 \} R_* \), and which thus should represent an optimal case for any possible magnetic spin-up into Keplerian orbit.

Figure 6 illustrates results of 2D MHD simulations for this case, using the approach and general model assumptions described in ud-Doula & Owocki (2002), but now extended to include field-aligned rotation (ud-Doula et al., 2008). The left panel shows that conditions at a time 90 ksec after introduction of the field do superficially resemble a magnetically torqued disk. Closer examination shows, however, that most of this equatorial compression does not have the appropriate velocity for a stable, stationary, Keplerian orbit. Thus, in just a few ksec of subsequent evolution, this putative “disk” becomes completely disrupted, characterized generally by infall of the material in the inner region, i.e. below the Keplerian radius \( R_K \), and by outflow in the outer region above this Keplerian radius. The right panel illustrates the irregular form of the dense compression at an arbitrarily chosen later time (390 ksec from the initial start). The arrows emphasize the flow divergence of the dense material both downward and upward from the Keplerian
4.4 A Rigidly Rotating Magnetosphere Model for Strongly Magnetic Hot Stars

The above MTD scenario was proposed to explain the Keplerian disks inferred from the characteristic Balmer line emission of Be stars. The general lack of rotational modulation in such Be-star line emission implies an overall axisymmetry that requires any magnetic field (which are not generally detected) producing a putative MTD would have to have a dipole axis closely aligned with the stellar rotation axis, as indeed was assumed in the above MHD simulations. By contrast, the so-called Bp stars do often exhibit clear rotational modulation in circumstellar emission lines, along with very strong magnetic fields (several kG) that are inferred to have a dipole axis that is tilted by some angle $\beta$ relative to the rotation axis.

For example, in the prototypical Bp star $\sigma$ Ori E, the magnetic field is estimated to have a dipole surface strength $B \sim 10^4$ G and tilt angle $\beta \approx 45^\circ - 70^\circ$, with a comparable observer's inclination $i = 45^\circ$ leading to modulation of Zeeman polarization on a rotation period of 1.19 day (Groote and Hunger, 1982). Coupled with a relatively low mass-loss rate ($\dot{M} \sim 10^{-10} M_\odot \text{ yr}^{-1}$), this implies an extremely strong magnetic confinement for the wind ($\eta_* \sim 10^5$). Unfortunately, direct MHD simulation of this case is severely complicated by the inherently 3-D nature associated with the nonzero tilt angle $\beta$, and by the extreme stiffness of the magnetic field. The latter implies a very high Alfvén speed, and thus very short Courant timestep, needed to preserve numerical stability within the explicit timestepping of the ZEUS code. Together these considerations make direct MHD
simulations of winds from Bp stars like σ Ori E impractical.

However, by considering the idealized limit of arbitrarily strong confinement ($\eta \to \infty$), it is possible to develop a quite intricate description of the *Rigidly Rotating Magnetosphere* (RRM) that is likely to form in such strongly magnetic, rotating Bp stars. In this limit, the field lines behave like rigid tubes, constraining the outflowing wind plasma along trajectories that are fixed by the *a priori* field geometry. With sufficiently rapid rotation, the outward centrifugal force, arising from the enforced corotation of the plasma, can support post-MCWS plasma at the tops of closed magnetic loops, in magnetohydrostatic configurations centered on the minima of the effective (centrifugal plus gravitational) potential along each field line. With the steady feeding of wind material from the star, these potential wells should gradually fill with cool plasma, forming a quasi-steady magnetosphere that co-rotates with the star. A strength of this RRM approach is its suitability to *arbitrary* field configurations, not just to the simple axisymmetric case of a rotation-axis-aligned dipole, to which MHD simulations have so far been restricted on grounds of computational tractability.

Application of the RRM formalism to an oblique-dipole model star, with parameters appropriate to σ Ori E, leads to a specific prediction of the accumulation
of wind material in two co-rotating circumstellar clouds, situated at the intersection between magnetic and rotational equators (Townsend and Owocki, 2004, see also Fig. 7). This prediction matches quite well the observationally inferred distribution of plasma proposed by (Groote and Hunger, 1982) and others. Using techniques originally developed for spectral synthesis from pulsating hot stars (e.g., Townsend, 1997; Townsend et al., 2004), one can calculate time-resolved Hα profiles for the RRM σ Ori E model (Townsend et al., 2005); as shown in Fig. 8, these synthetic profiles exhibit a remarkable degree of agreement with the corresponding observations. A similar level of agreement is found between the predicted optical and IR photometric behavior, and that observed by Hesser et al. (1977).

4.5 A Rigid-Field Hydrodynamics Approach

Recent efforts by Townsend et al. (2007) have extended this semi-analytic RRM analysis into a Rigid Field HydroDynamics (RFHD) approach in which the full hydrodynamical evolution is simulated separately along each of many distinct rigid field line. These separate simulations can then be stitched together to provide a 3D dynamical model of the evolving magnetosphere. Rather than just focussing on the hydrostatic stratification of material that has settled along the accumulation surface, this provides a dynamical description of the filling up of this accumulation surface by the stellar wind flux from the stellar surface. The difference between the RFHD vs. RRM approaches is thus somewhat analagous to modeling the filling up of a glass of water, instead of just describing the ultimate settling of that water within the glass. However, since for the RFHD case the filling is via a supersonic wind, the impact onto the accumulation surface generally involves a strong shock transition, with associated heating to temperatures up to ca. $10^8$ K, high enough again to emit quite hard X-rays, with energies of several keV or more. But the results also confirm that, once this material cools back to temperatures of order the stellar effective temperature (typically several time $10^4$ K), it does indeed settle into a hydrostatic stratification centered on the accumulation surface. This thus provides a quite reassuring independent confirmation of the general validity of the basic RRM analysis for this cooler disk material.

4.6 A Centrifugal Breakout Mechanism for X-ray Flaring

In routine X-ray observations of σ Ori E by the Rosat, XMM, and Chandra satellites, two of the three ca. day-long exposures showed clear evidence for a quite strong, hard, X-ray flare (Groote and Schmitt, 2004). Such X-ray flaring from an early-type star is unusual and unexpected, and so was initially attributed instead to an unseen cool companion star (Sanz-Forcada et al., 2004), for which flaring is commonly associated with magnetic reconnection heating arising from the activity of a convective magnetic dynamo. However, the strength and hardness of these flares make it unlikely that they could be produced within the inherent length contraints for magnetic loops from such a cool star (D. Mullan, p.c.), which then suggests that they might in fact be associated with either the Bp star or its
Fig. 9. MHD simulations of a Centrifugal Breakout model for X-ray flaring, showing the logarithmic temperature $T$ in a meridional plane. Left: at a time 190 ksecs, the centrifugal force acting on dense material in the equatorial plane has drawn the magnetic field out into a long, narrow neck. Middle: at a time 220 ksecs, the stressed magnetic field has reconnected, heating material in the outer regions of the equatorial plane to $T \sim 10^8$ K. Right: at a time 240 ksecs, the reconnected field has snapped back toward the star, producing further heating.

The immediate circumstellar environment. Similar conclusions can be reached regarding a flare detected in Chandra ACIS-I observations of the young O9.5Vpe star $\theta^2$ Ori A (see, for example, Feigelson et al., 2002). This represents a new instance of X-ray flare production, which – with the absence of deep sub-photospheric convection zones in hot stars – appears challenging to explain purely in terms of the traditional mechanisms operative in cooler stars.

Fortunately, the above RRM model can provide a quite natural alternative explanation for the flaring seen in these magnetic hot stars. The steady accumulation of plasma in a RRM cannot continue indefinitely; eventually, circumstellar densities reach levels where the outward centrifugal force must overwhelm the inward magnetic tension forces, leading to the breakout of plasma from the magnetosphere in a direction perpendicular to the rotation axis. Townsend and Owocki (2004) present a simple analysis of this breakout for the case of $\sigma$ Ori E; they suggest that the largest-scale evacuations can be expected every $\sim 100$ yr, but they also anticipate a whole hierarchy of breakout events extending down to much shorter timescales. During a breakout, the magnetic field lines become so drawn out by the ejected plasma that they snap and then reconnect closer to the star. The energy release associated with this reconnection, and its subsequent dissipation via radiative cooling, represents a strong candidate for the generation of the X-ray flares.

Initial simulations of this new Centrifugal Breakout mechanism for X-ray production appear quite encouraging (ud-Doula et al., 2006). Although it is not yet feasible to conduct MHD simulations at a confinement parameter $\eta_s \sim 10^7$ appropriate to $\sigma$ Ori E (for the reasons noted previously, relating to the Courant condition), fig. 9 shows results for a moderately confined ($\eta_s \sim 600$) case, rotating at half the critical rate ($W = 1/2$) at which the surface gravitational and centrifugal...
gal forces would balance along the equator. The simulations reveal that, after the initial formation of a small rigidly rotating magnetosphere close to the star (as predicted by the RRM model), a semi-regular sequence of breakout events occurs, whereby field lines are pulled out into long, narrow loops, before snapping back toward the star (cf. middle and right panels of fig. 9). The energy released by the reconnection is sufficient to heat nearby material to temperatures $T \sim 10^8\,\text{K}$, high enough to produce the hard ($\gtrsim 2\,\text{keV}$) components of the flares observed in $\sigma$ Ori E and $\theta^2$ Ori A.

4.7 MHD Parameter Study for Rotation and Magnetic Confinement

ud-Doula et al. (2008) have carried out an extensive parameter study of MHD simulations for a broad range in both the magnetic confinement $\eta_*$ and the rotation ratio $W$. The results provide an intriguing glimpse into the complex, time-dependent behavior resulting from the competition among wind driving, magnetic channeling, and the centrifugal effects of rotation. A key result is that there is really no true steady state possible, since the secular buildup of material in the disk must eventually lead to an episodic material breakout once the centrifugal forces overwhelm the finite magnetic tension. They also allow one to examine in detail the nature of this build-up and dissipation of mass in an RRM disk, and how this varies with the changes in the rotation rate and magnetic confinement. To facilitate illustration of these competing processes, ud-Doula et al. (2008) define a radial mass distribution of the disk, computed at each radius $r$ in terms of the mass within some specified co-latitude range about the equator,

$$\frac{dm_e(r, t)}{dr} \equiv 2\pi r^2 \int_{\pi/2 - \Delta \theta/2}^{\pi/2 + \Delta \theta/2} \rho(r, \theta, t) \sin \theta \, d\theta . \quad (4.1)$$

To isolate the disk but not miss too much disk material during various oscillations about the equator, we choose a narrow, but not-too-limited range $\Delta \theta = 10^\circ$. Figure 10 shows schematically how this is computed.

Figure 11 compares the radius and time evolution of the equatorial mass, $dm_e/dr$, for an mosaic of models with various $\eta_*$ and $W$. The comparison provides a global overview of how the equatorial mass evolution is affected by changes in confinement and rotation.

For weak rotation and confinement cases in the lower left panels, material generally escapes outward without much infall, with only a modest rotational enhancement in mass loss. But most other models again show a complex competition between infall and breakout, with the latter always being less frequent and stronger.

In particular, this complex combination of infall and breakout also dominates the $R_A \approx R_K$ transition models, i.e. the ones here with $\log \eta_* = 1/2$ and $W = 1/2$ or $\log \eta_* = 3/2$ and $W = 1/4$. Such models might seem optimally fine-tuned to propel material into Keplerian orbit, and yet they show no apparent tendency for material to accumulate into the extended, Keplerian disk envisioned in the MTD scenario suggested by Cassinelli et al. (2002). The lack of a sharp outer cutoff in
Fig. 10. Schematic diagram illustrating the computation of the radial mass distribution $dm_e/dr$ (see eqn. 4.1), within a cone angle of $\Delta \theta = 10^\circ$ centered on the magnetic equator.

Fig. 11. Logarithm of radial distribution of equatorial disk mass, $dm_e/dr$ vs. radius and time, for a mosaic of models with magnetic confinements $\log(\eta_e) = 1/2, 1, 3/2, 2, 5/2$ and 3 (columns from left) and rotations $W = 0, 1/4, 1/2$ (rows from bottom). The horizontal lines indicate the Alfvén radius $R_A$ (solid) and the Kepler radius $R_K$ (dashed). The shading represents $\log(dm_e/dr)$ in units of $M_\odot/R_*$ over a range from $-10$ (white) to $-7$ (black).

the large-scale dipole field makes it incompatible with the shear of a Keplerian disk, and without the closed loops that hold down a rigid disk in the strong-confinement limit, material is propelled outward to escape, rather than into a stable Keplerian orbit.

As expected, accumulation into such a rigid-body disk is strongest for the fastest rotation, and strongest confinement, as shown by models at the upper right. For strong confinement but slow or no rotation, the material infall comes from a
greater height, set by the closure radius, which increases roughly as \( R_c \sim \eta_*^{1/4} \).

This larger inflow height seems also to lead to a somewhat longer inflow timescale. Likewise, the breakout timescale also seems to increase for models with stronger confinement parameter \( \eta_* \), but not quite in the linear proportion that might be suggested by a simple “breakout” analysis (Townsend and Owocki, 2004; ud-Doula et al., 2008). Note also that the \( W = 1/2 \) rotation model with the strongest confinement, \( \eta_* = 1000 \), is relatively stable, without the repeated equatorial inflow events seen in other models. Instead of the extensive north-south disk oscillations seen in other models, in this case the variations of the equatorial disk are mostly symmetric about the equator, and thus do not induce as much “spillage” back onto the star. The recent analysis of “Rigid-Field Hydrodynamics” (RFHD) models by Townsend et al. (2007) show that both types of oscillation modes are allowed, with the one dominating in simulations depending on subtle details of the excitation processes.

But overall, it seems that the basic principles gleaned from the detailed study of the standard, strong-confinement case can be quite logically generalized to understand the trends in properties seen from this mosaic of models spanning a broad range of rotation and confinement parameters.

### 4.8 Summary for Massive-Star Magnetospheres

The results here demonstrate that magnetic fields in hot stars with moderate to large confinement parameters (\( \eta_* > 1 \)) can have a substantial effect in channeling their radiatively driven stellar wind. In the example of the recently detected, moderate-strength magnetic field from the slowly rotating O7V star \( \theta_1 \) Ori C, the MHD simulations of magnetically confined wind shocks provide a quite good match to the observed X-ray properties. The addition of rotation can spin up the wind, but does not lead to Magnetically Torqued Disk that could explain the orbiting circumstellar material inferred from the characteristic Balmer emission of Be stars. However, in rotating Bp stars with very strong fields, the channeling of the wind can feed a rigid-body disk or clouds, which can be well modeled within a semi-analytic RRM formalism that provides a quite good match to the rotational modulations in line and continuum observations. Moreover, the eventual centrifugal breakout of such material suggests a new mechanism for magnetic reconnection heating to explain observed hard X-ray flares in several such Bp stars.

While the present discussion has focused mostly on model development for cases with relatively large, detectable fields (\( B > 100 \text{G} \)), it also seems likely that the base perturbations associated with more moderate, still undetected fields could also play a role in the semi-regular variability commonly detected in the UV wind lines of hot stars. To model such modulations from fields that are not symmetric about the stellar rotation axis, future simulations will need to be extended to 3-D. If such MHD models can provide a good match to the observed variability, it would support the notion that, contrary to the classical picture, hot-star magnetic fields are not limited to a few cases or peculiar classes, but are a common, perhaps even ubiquitous, feature. This has potentially quite broad implications, requiring
for example that there must be a mechanism for stellar field generation that is quite distinct from the usual convective dynamo operating in the Sun and other cool stars. It might even require reappraisal of our basic assumptions regarding the envelope and interior of early-type stars. The study here of magnetic channeling of hot-star winds represents another illustration of the complex but broad significance of magnetic fields for astrophysical plasmas.

References


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