Radiatively Driven Stellar Winds from Hot Stars

A stellar wind is the continuous, supersonic outflow of matter from a star. Among the most massive stars—which tend also to be the hottest and most luminous—the winds can be very strong, with important consequences both for the star's own evolution as well as for the surrounding interstellar medium. Such hot-star winds are understood to be driven by the pressure of the star's emitted radiation.

Our Sun has a SOLAR WIND, but it is so tenuous and transparent that it would be difficult to detect directly from a more distant star. However, many stars have winds that are dense enough to be opaque at certain wavelengths of the star's radiation, and this makes it possible to study them remotely through careful interpretation of the observed stellar spectra. The winds from massive, hot stars—with surface temperatures above about 10 000 K—form a well-defined class, distinct from the winds from cooler stars, and characterized by the central role of the star's own radiation in driving the mass outflow. The high temperature of such stars means that they have a very high surface brightness. Because light carries momentum as well as energy, this high surface brightness imparts a force to the atoms that scatter the light. At the level of the star's atmosphere where this force exceeds the inward force of the stellar gravity, material is accelerated upward and becomes the stellar wind.

An important aspect of this radiative driving process is that it stems mostly from *line* scattering, wherein an electron is shuffled between two discrete, bound energy levels of an atom. In a static medium such scattering is confined to radiation with a photon energy near the energy difference between the levels, corresponding to a range of wavelengths near a distinct, line-center value. However, in an accelerating stellar wind flow, the Doppler effect shifts the resonance to increasingly longer wavelengths, allowing the line scattering to sweep gradually through a much broader portion of the stellar spectrum. This gives the dynamics of such winds an intricate feedback character, in which the radiative driving force that accelerates the stellar wind depends itself on that acceleration.

Overview and comparison with the solar wind

The general concept that stars can sometimes lose or eject matter is apparent from some spectacular, catastrophic examples, such as nova or supernova explosions. However, the specific notion of a *continuous*, quiescent-phase stellar wind outflow stems largely from discovery, during the 1950s and 1960s, of the *solar* wind. The Sun is a relatively low-mass, cool star with a surface temperature about 6000 K, but curiously its wind arises from pressure expansion of the very hot, million kelvin solar corona, which is somehow superheated by the mechanical energy generated from convection in the Sun's subsurface layers. By contrast, high-mass stars with much higher surface temperatures (10 000–100 000 K) are thought to lack the strong convection zone needed to heat a circumstellar corona. Their stellar winds thus remain at temperatures comparable with the star's surface, and so lack the very high gas pressure needed to drive an outward expansion against the stellar gravity. However, such hot stars have a quite high radiative flux, since by the Stefan–Boltzmann law this scales as the fourth power of the surface temperature. It is the pressure of this *radiation* (not of the gas itself) that drives the wind expansion.

The typical flow speeds of hot-star winds—up to about 3000 km s^{-1}—are a factor of a few faster than the 400–700 km s^{-1} speed of the solar wind. However, the inferred mass loss rates of hot stars greatly exceed—by up to a factor of a billion—that of the Sun. At the Sun's current rate of mass loss, about $10^{-14} M_\odot$ yr^{-1}, its mass would be reduced by only ~0.01% during its entire characteristic lifespan of 10 billion (10^{10}) years. By contrast, even during the much shorter, few million (10^6) year lifetime typical for a massive star, its wind mass loss at a rate of up to $10^{-5} M_\odot$ yr^{-1} can substantially reduce, by a factor of 2 or more, the original stellar mass of a few times $10 M_\odot$. Indeed, massive stars typically end up as 'WOLF–RAYET' STARS, which often appear to have completely lost their original envelope of hydrogen, leaving exposed at their surface the elements such as carbon, nitrogen and oxygen that were synthesized by nuclear processes in the stellar core. (Much of this cumulative mass loss might also occur during relatively brief 'superwinds' or epsisodic ejections as a 'luminous blue variable'.) In addition to directly affecting the star's own evolution, hot-star winds often form 'wind-blown bubbles' in nearby interstellar gas. Overall they represent a substantial contribution to the energy, momentum and chemical enrichment of the interstellar medium in the Milky Way and other galaxies.

Wind detection through asymmetric, P-Cygni line profiles

The high density of hot-star winds means that the expanding material is itself opaque, or *optically thick*, to scattering in many atomic spectral-line transitions. When observed as part of a stellar spectrum, radiation that is line scattered within the wind develops a characteristic line profile, known as a 'P-Cygni profile', after the star P-Cygni in which its significance as a signature of mass outflow was first broadly recognized.

Figure 1 illustrates the formation of a P-Cygni line profile. Wind material approaching an observer within a column in front of the star has its line resonance blueshifted by the Doppler effect. Thus scattering of stellar radiation out of this direction causes an absorption trough, or reduction in the observed flux, on the blue side of the line profile. However, from the lobes on either side of this absorption column, wind material can also scatter radiation *toward* the observer. Since this can occur from either the approaching or the receding hemisphere, this scattered radiation can be either blueshifted or redshifted. The associated extra flux seen by the observer thus occurs as a symmetric emission component on both sides of

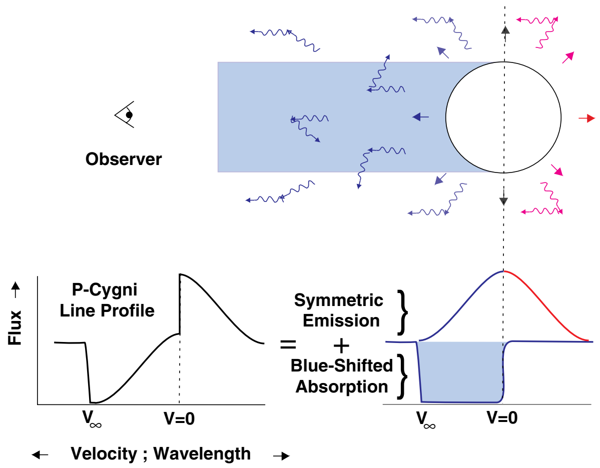

Figure 1. Schematic illustration of the formation of a P-Cygni type line profile in an expanding wind outflow.

the line center. Combined with the reduced blue-side flux, the overall profile has a distinctly asymmetric form, with apparent net blueward absorption and redward emission. Moreover, the wavelength of the blue edge of the absorption provides a quite unambiguous measure of the asymptotic wind speed, v_∞.

The radiative force from electron scattering

A radiative force results from the material interception of radiative momentum. A particularly simple case is scattering by free electrons, which is a 'gray', or frequency-independent, process. Since gray scattering cannot alter the star's total luminosity L, the radiative energy flux at any radius r is simply given by $L/4\pi r^2$. This corresponds to a radiative momentum flux of $L/4\pi r^2 c$, where c is the speed of light. The material interception of this flux is characterized by an interaction cross section per unit mass, κ, which is formally named the 'mass absorption coefficient' but more succinctly is also commonly called the 'opacity'. For electron scattering in an ionized medium, the opacity is simply a constant given by $\kappa_e = \sigma_e/\mu_e$, where σ_e ($=0.66 \times 10^{-24}$ cm^2) is the classical Thompson cross section, and the mean atomic mass per free electron is $\mu_e = 2m_H/(1+X)$, with m_H and X the mass and mass fraction of hydrogen. This works out to a value $\kappa_e = 0.2(1+X) = 0.34$ cm^2 g^{-1}, where the latter result applies for the standard (solar) hydrogen mass fraction $X = 0.72$. The product of this opacity and the radiative momentum flux yields the radiative acceleration (force per unit mass) from free-electron scattering,

$$g_e(r) = \frac{\kappa_e L}{4\pi r^2 c}.$$

It is of interest to compare this with the star's gravitational acceleration, given by GM/r^2, where G is the gravitation constant and the M is the stellar mass. Since both accelerations have the same $1/r^2$ dependence on radius, their ratio is spatially constant, fixed by the ratio of luminosity to mass,

$$\Gamma_e = \frac{\kappa_e L}{4\pi G M c}.$$

This ratio, sometimes called the Eddington parameter, thus has a characteristic value for each star. For the Sun it is very small, of order 2×10^{-5}, but for hot, massive stars it is often within a factor of 2 below unity. As noted by Eddington, electron scattering thus represents a basal radiative acceleration that effectively counteracts the stellar gravity. The limit $\Gamma_e \to 1$ is known as the Eddington limit, for which the star would become gravitationally unbound.

It is certainly significant that hot stars with strong stellar winds have Γ_e only a factor of 2 or so below this limit, since it suggests that only a modest additional opacity could succeed in fully overcoming gravity to drive an outflow. However, it is important to realize that a stellar wind represents the outer envelope outflow from a nearly static, gravitationally bound base, and as such is not consistent with an entire star exceeding the Eddington limit. Rather the key requirement for a wind is that the driving force increase naturally from being smaller to larger than gravity at some radius near the stellar surface. We shall now see that the force from line scattering is ideally suited for just such a spatial modulation.

The Doppler-shifted resonance of line scattering

When an electron is bound into one of the discrete energy levels of an atom, its scattering of radiation is primarily with photons of just the right energy to induce shuffling with another discrete level. The process is called line scattering, because it often results in the appearance of narrowly defined lines in a star's energy spectrum. At first glance, it may seem unlikely that such line scattering could be effective in driving mass loss, simply because the opacity only interacts with a small fraction of the available stellar flux. However, there are two key factors that work to make line scattering in fact the key driving mechanism for hot-star winds.

The first is the resonant nature of line scattering. The binding of an electron into discrete energy levels of an atom represents a kind of resonance cavity that can greatly amplify the interaction cross section with photons of just the right energy to induce transition among the levels. The effect is somewhat analogous to blowing into a whistle versus just into open air. Like the sound of a whistle, the response occurs at a well-tuned frequency and has a greatly enhanced strength. Relative to free-electron scattering, the overall amplification factor for a broad-band, untuned radiation source is set by the quality of the resonance, $Q \approx \nu_0/A$, where ν_0 is the line frequency and A is the decay rate of the excited state. For quantum mechanically allowed atomic transitions, this can be very large, of order 10^7. Thus, even though only a very small fraction ($\sim 10^{-4}$) of electrons in a hot-star atmosphere are bound into atoms, illumination of

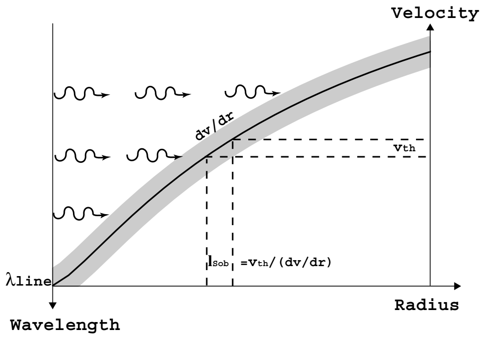

Figure 2. Doppler-shifted line resonance absorption in an accelerating flow. Photons with a wavelength just shortward of a line propagate freely from the stellar surface until redshifted, within a reference frame moving with the wind outflow velocity, into a narrow line resonance, whose width is set by the thermal broadening of the line, as represented here by the shading.

these atoms by an unattenuated (i.e. optically thin), broad-band radiation source would yield a collective line force that exceeds that from free electrons by about a factor $\bar{Q} \approx 10^7 \times 10^{-4} = 1000$. For stars within a factor of 2 of the free-electron Eddington limit, this implies that line scattering is capable, in principle, of driving material outward with an acceleration on the order of a thousand times the inward acceleration of gravity.

In practice, of course, this does not normally occur, since any sufficiently large collection of atoms scattering in this way would quickly block the limited flux available within just the narrow frequency bands tuned to the lines. Indeed, in the static portion of the atmosphere, the flux is greatly reduced at the line frequencies. Such line 'saturation' keeps the overall line force quite small, in fact well below the gravitational force, which thus allows the inner parts of the atmosphere to remain gravitationally bound.

This, however, is where the second key factor, the Doppler effect, comes into play. In the outward-moving portions of the outer atmosphere, the Doppler effect redshifts the local line resonance, effectively desaturating the lines by allowing the atoms to resonate with relatively unattenuated stellar flux that was initially at slightly higher frequencies. By effectively sweeping a broader range of the stellar flux spectrum, this makes it possible for the line force to overcome gravity and accelerate the very outflow it itself requires. As quantified within the CAK wind theory described later, the amount of mass accelerated adjusts such that the self-absorption of the radiation reduces the overall line driving to being just somewhat (not a factor of 1000) above what is needed to overcome gravity.

The Sobolev approximation for line scattering in an accelerating flow

The late Russian astrophysicist V V Sobolev developed an extremely useful approach for treating line scattering in such a rapidly accelerating flow. As illustrated in figure 2, he noted how radiation emitted from the star at a wavelength somewhat blueward of a line propagates freely until it is redshifted by the accelerating flow into a local line resonance. For the usual case that the broadening of the line is set by the ion thermal speed $v_{\rm th}$, the geometric width of this resonance is about a Sobolev length, $l_{\rm Sob} \equiv v_{\rm th}/({\rm d}v/{\rm d}r)$. In a supersonic flow, this Sobolev length is of order $v_{\rm th}/v \ll 1$ smaller than a typical flow variation scale, like the density–velocity scale length $H \equiv |\rho/({\rm d}\rho/{\rm d}r)| \approx v/({\rm d}v/{\rm d}r)$.

The nearly homogeneous conditions within such resonance layers imply that the key parameters of the line scattering can be described in terms of strictly *local* conditions at any radius. In particular, the total optical depth of radiation passing through the line resonance—which normally requires evaluation of a nonlocal spatial integral—can in this case be well approximated simply in terms of the local density and velocity gradient,

$$\tau = \frac{\kappa \rho v_{\rm th}}{{\rm d}v/{\rm d}r} \tag{1}$$

where κ is the line opacity. This allows a localized solution for how much the flow absorption reduces the illumination of the line resonance by the stellar flux and leads to a simple, general expression for the line acceleration,

$$g_{\rm line} \approx g_{\rm thin} \frac{1 - {\rm e}^{-\tau}}{\tau}. \tag{2}$$

In the optically thin limit $\tau \ll 1$, the line acceleration reduces to a form similar to the electron scattering case,

$$g_{\rm thin} \equiv \frac{\kappa v_{\rm th} \nu_0 L_\nu}{4\pi r^2 c^2} \approx \frac{\kappa}{\kappa_{\rm e}} \frac{v_{\rm th}}{c} g_{\rm e} \tag{3}$$

wherein the last approximate equality applies the usual assumption that the line frequency ν_0 is near the peak of the stellar luminosity spectrum L_ν, so that $\nu_0 L_\nu \approx L$. In the opposite limit of an optically thick line with $\tau \gg 1$, there results a quite different form,

$$g_{\rm thick} \approx \frac{g_{\rm thin}}{\tau} = \frac{L}{4\pi r^2 \rho c^2} \frac{{\rm d}v}{{\rm d}r} = \frac{L}{\dot{M} c^2} v \frac{{\rm d}v}{{\rm d}r} \tag{4}$$

where the last equality uses the definition of the wind mass loss rate, $\dot{M} \equiv 4\pi \rho v r^2$.

A key result here is that the optically thick line force is independent of the opacity and instead varies in proportion to the velocity gradient ${\rm d}v/{\rm d}r$. The basis of this result is illustrated by figure 2, which shows that the local rate at which stellar radiation is redshifted into a line resonance depends on the slope of the velocity. By Newton's famous equation of motion, a force is normally understood to *cause* an acceleration. However, here we see that an optically thick line force also *depends* on the wind's advective rate of acceleration, $v\,{\rm d}v/{\rm d}r$.

Derivation of direct absorption force for a single line

Let us derive this key property of line driving in more quantitative terms. Under the simplifying approximation that the stellar radiation flux is purely radial (as from a central point source), the force per unit mass associated with direct absorption by a single line at a radius r is given by

$$g_{\rm line}(r) = g_{\rm thin} \int_{-\infty}^{\infty} {\rm d}x\, \phi[x - v(r)/v_{\rm th}]\, {\rm e}^{-t(x,r)}. \qquad (5)$$

The integration is over a scaled frequency $x \equiv (\nu/\nu_0 - 1)\, c/v_{\rm th}$, defined from a line center frequency, ν_0, in units of the frequency broadening associated with the ion thermal motion, $v_{\rm th}$. The integrand is weighted by the line profile function $\phi(x)$, which for thermal broadening typically has the Gaussian form $\phi(x) \sim {\rm e}^{-x^2}$. The exponential reduction takes account of absorption, as set by the frequency-dependent optical depth to the stellar surface radius R_*,

$$t(x, r) \equiv \int_{R_*}^{r} {\rm d}r'\, \kappa\rho(r')\phi[x - v(r')/v_{\rm th}].$$

A crucial point in evaluating this integral is that, in a supersonic wind, the variation of the integrand is dominated by the velocity variation within the line profile. As noted above, this variation has a scale given by the Sobolev length $l_{\rm Sob} \equiv v_{\rm th}/({\rm d}v/{\rm d}r)$, which is smaller by a factor $v_{\rm th}/v$ than the competing density–velocity scale, $H \equiv |\rho/({\rm d}\rho/{\rm d}r)| \approx v/({\rm d}v/{\rm d}r)$. A key step in the Sobolev approximation is thus to recast this spatial integration as an integration over the comoving frame frequency $x' \equiv x - v(r')/v_{\rm th}$,

$$\begin{aligned} t(x, r) &= -\int_{x'(R_*)}^{x'(r)} \frac{{\rm d}x'\, v_{\rm th}}{{\rm d}v/{\rm d}r'} \kappa\rho(x')\,\phi(x') \\ &\approx \tau(r) \int_{x - v(r)/v_{\rm th}}^{\infty} {\rm d}x'\, \phi(x') \end{aligned} \qquad (6)$$

where the latter approximation uses the assumption that $v(r) \gg v_{\rm th}$ to formally extend the surface frequency $x'(R_*)$ to infinity relative to the local resonance $x'(r)$. The quantity $\tau(r)$, which is just the local Sobolev optical thickness defined in equation (1), arises as a collection of spatial variables that are assumed to be nearly constant over the Sobolev resonance zone and thus can be extracted outside the integral.

Finally, a remarkable, extra bonus from this approximation is that the resulting optical depth (6) now has precisely the form needed to allow analytical evaluation of the line force integral (5), yielding directly the general expression given in equation (2).

The mass loss rate from inertial force balance

We can actually estimate the wind mass loss rate from a simple consideration of how this line force overcomes the wind inertia (for now ignoring gravity). For a single optically thick line near the peak of the stellar flux spectrum, the resulting steadystate flow acceleration is given by

$$v\frac{{\rm d}v}{{\rm d}r} = g_{\rm line} \approx \frac{L}{\dot{M}c^2} v \frac{{\rm d}v}{{\rm d}r}.$$

Since the acceleration cancels from both sides, this simply implies $\dot{M} \approx L/c^2$, or that the total mass loss of the wind is roughly equal to just the mass loss associated with the stellar luminosity. In practice, hot-star winds are driven by not just one but many optically thick lines. As long as these are independent of each other, the total resulting mass loss simply accumulates in proportion to the total number of thick lines,

$$\dot{M} \approx N_{\rm thick} \frac{L}{c^2}. \qquad (7)$$

For a wind with terminal speed v_∞, each line sweeps through about a fraction v_∞/c of the spectrum. This means that there can at most be about $N_{\rm thick} \approx c/v_\infty$ nonoverlapping thick lines spread throughout the spectrum. This implies a single-scattering limit for the mass loss,

$$\dot{M} v_\infty < \frac{L}{c}.$$

The winds from stars of spectral types O and B are generally within this limit, but winds from Wolf–Rayet (WR) stars can exceed it by a factor of 10 or more. As further discussed below, the driving of such WR winds thus requires a more intricate, multiline scattering process.

The CAK theory for line-driven winds

In practice, the number of thick lines depends itself on the mass loss rate and so is not known *a priori*. Self-consistent solution of the wind properties is possible, however, through the formalism developed by Castor, Abbott and Klein (CAK). A key simplification is to approximate the flux-weighted number distribution of lines as a power law in the line opacity κ,

$$\frac{{\rm d}N}{{\rm d}\kappa} = \frac{1}{\kappa_0}\left(\frac{\kappa}{\kappa_0}\right)^{\alpha - 2}$$

where the CAK power-law index satisfies $0 < \alpha < 1$, and κ_0 is a normalization constant defined such that $\kappa_0\, {\rm d}N/{\rm d}\kappa \equiv 1$. (The latter is related to the commonly defined CAK constant k by $k = \Gamma(\alpha)(v_{\rm th}/c)(\kappa_0/\kappa_{\rm e})^{1-\alpha}/(1 - \alpha)$, where Γ denotes the complete gamma function.) The cumulative force from this line ensemble can be obtained by an integration of the single-line result (2) over this opacity distribution. This yields a kind of 'geometric mean' between the optically thin and thick forms (equations (3) and (4)) for a single line,

$$g_{\rm cak} = \frac{g_{\rm thin,o}}{\tau_{\rm o}^{\alpha}}. \qquad (8)$$

Here $\tau_{\rm o}$ and $g_{\rm thin,o}$ are respectively the Sobolev optical thickness and the optically thin acceleration for a single,

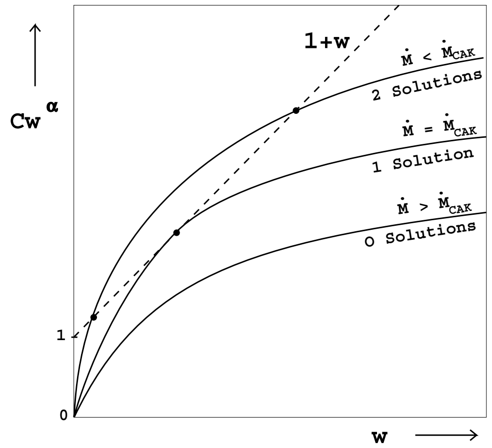

Figure 3. Graphical solutions of the dimensionless equation of motion (9).

spectral-peak line of opacity κ_o. This normalization opacity is given roughly by the line resonance amplification factor discussed previously,

$$\frac{\kappa_o v_{th}}{c} \approx \bar{Q}\kappa_e \approx 10^3 \kappa_e$$

which thus implies that $g_{thin,o} \approx 10^3 g_e$, as noted earlier. In this context, $\bar{Q}$ can be thus thought of an alternative, dimensionless way to specify the opacity normalization.

Ignoring the generally unimportant contribution of gas pressure, the steady-state equation of motion simply requires that the wind acceleration must equal the line acceleration minus the inward acceleration of gravity,

$$v\frac{dv}{dr} = g_{cak} - \frac{GM(1-\Gamma_e)}{r^2}$$

wherein we have also taken into account the effective reduction of gravity by the free-electron scattering factor Γ_e.

Defining the flow acceleration in units of the gravity as

$$w \equiv r^2 v v' / GM(1-\Gamma_e)$$

the equation of motion can be rewritten in the simple, dimensionless form,

$$w = C\, w^\alpha - 1 \qquad (9)$$

where the constant is given by

$$C = \left(\frac{\bar{Q}\Gamma_e}{1-\Gamma_e}\right)^{1-\alpha} \left(\frac{L}{\dot{M}c^2}\right)^{\alpha}.$$

Figure 3 illustrates the graphical solution of this dimensionless equation of motion for various values of the constant C. For fixed stellar and opacity distribution parameters, this corresponds to assuming various values of the mass loss rate $\dot{M}$. For high $\dot{M}$ there are no solutions, while for low $\dot{M}$ there are two solutions. The two limits are separated by a critical case with one solution—corresponding to the *maximal* mass loss rate—for which the function Cw^α intersects the line $1 + w$ at a tangent. For this critical case, the tangency requirement implies $\alpha C_c w_c^{\alpha-1} = 1$, which together with the original equation (9) yields the critical conditions $w_c = (\alpha/1-\alpha)$ and $C_c = 1/(\alpha^\alpha(1-\alpha)^{1-\alpha})$.

Since equation (9) has no explicit spatial dependence, these conditions hold at all radii. By spatial integration of the critical acceleration w_c from the surface radius R_*, we thus obtain the CAK velocity law,

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^{1/2} \qquad (10)$$

where the terminal speed is given by $v_\infty = v_{esc}[\alpha/(1-\alpha)]^{1/2}$, where v_{esc} is the effective surface escape speed. Likewise, the associated critical value of C_c defines the maximal CAK mass loss rate

$$\dot{M}_{cak} = \alpha \left(\frac{(1-\alpha)\bar{Q}\Gamma_e}{1-\Gamma_e}\right)^{(1-\alpha)/\alpha} \frac{L}{c^2}. \qquad (11)$$

Comparison with equation (7) shows that the first two factors in equation (11) now provide an explicit expression for the number of thick lines.

These results strictly apply only under the idealized assumption made above that the stellar radiation is radially streaming from a point source. If one takes into account the finite angular extent of the stellar disk, then the mass loss is slightly reduced (typically by about a factor of 2), and the velocity law becomes slightly flatter (approximated by replacing the exponent 1/2 in equation (10) by a value $\beta \approx 0.8$), with a somewhat higher terminal speed ($v_\infty \approx 3v_{esc}$).

Finally, another simplifying assumption of this analysis is that the line opacities are spatially constant, which implies a fixed wind ionization. The effect of a radial change in ionization can be approximately taken into account by correcting the CAK force (8) by a factor of the form $(n_e/W)^\delta$, where n_e is the electron density, $W \equiv 0.5[1-(1-R_*/r)^{1/2}]$ is the radiation 'dilution factor', and the exponent has a typical value $\delta \approx 0.1$. This factor introduces an additional density dependence to that already implied by the optical depth factor $1/\tau_o^\alpha$ given in equation (8). Its general effect on the mass loss can be roughly accounted with the simple substitution $\alpha \to \alpha' \equiv \alpha - \delta$ in the power exponents of the CAK mass loss scaling law (11). The general tendency is to moderately increase $\dot{M}$ and accordingly to somewhat decrease the wind speed.

The wind momentum–luminosity relation

An important success of these CAK scaling laws is the theoretical explanation they provide for an empirically observed 'wind momentum–luminosity' (WML) relation. Combining the CAK mass loss law (11) together with the scaling of the terminal speed v_∞ with the effective escape speed $v_{esc} = [2GM(1-\Gamma)/R]^{1/2}$, we obtain a WML relation of the form,

$$\dot{M} v_\infty R_*^{1/2} \sim L^{1/\alpha'} Q^{1/\alpha'-1}$$

wherein we have neglected a residual dependence on $M(1-\Gamma)$ that is generally very weak for the usual case that α' is near 2/3. Stellar mass loss rates can be routinely derived by fits to the observed Balmer (Hα) line emission, and wind terminal speeds can be inferred from the blue edge of observed P-Cygni profiles. Combined with spectroscopic estimates of the stellar luminosity and radius (the latter of which in any case enters only weakly as a square root), empirical fits for a large sample of galactic OB supergiants have demonstrated a remarkably tight agreement with a single WML relation. These fits give a luminosity slope that implies $\alpha' \approx 0.57$ and an overall normalization that is quite consistent with the expectation that $\bar{Q} \approx 10^3$.

Less extensive fits have also been derived for OB stars in the Magellanic clouds. These are each separately consistent with a single WML relation, but with a lower normalization, especially for the small cloud. This probably just reflects the weaker radiative driving from the lower metallicity Z, since the line opacity normalization varies in direct proportion to the metallicity, $\bar{Q} \sim Z$. There are currently efforts toward carefully calibrating this WML relation and its dependence on metallicity. The ultimate goal is to apply observationally inferred wind parameters of luminous hypergiant stars in external galaxies as an alternative 'standard candle' for extragalactic distance measurements. In this way hot-star winds may play a role in calibrating the distance scale of the universe.

The Wolf–Rayet wind 'momentum problem'

WR STARS are evolved, massive, hot stars for which the cumulative mass loss has led to depletion of the original hydrogen envelope. They typically show broad wind emission lines of elements such as carbon, nitrogen and/or oxygen that are the products of core nucleosynthesis. Overall, observations indicate that WR winds are especially strong, and even optically thick to continuum scattering by electrons. Notably, inferred WR wind momenta $\dot{M}v_\infty$ are generally substantially higher than for OB stars of comparable luminosity, placing them well above the OB star line in the WML relation. In fact, the inferred ratio of the wind momentum to that of stellar luminosity, $\eta \equiv \dot{M}v_\infty/(L/c)$, is typically much above unity, sometimes as high as $\eta = 10$–50.

This last property has generally been cast as the WR wind 'momentum problem', sometimes with the implication that it implies that WR winds cannot be radiatively driven. In fact, it merely means that, unlike for OB stars, WR winds cannot be treated in the standard, single-scattering formalism, which assumes optically thick lines do not overlap within the wind. However, momentum ratios above unity can, in principle, be achieved by multiple scattering between overlapping thick lines with a velocity unit frequency separation $\Delta v < v_\infty$. Indeed, if the spectral distribution of lines is 'gray'—i.e. spread randomly through the spectrum without substantial gaps or bunches—then the momentum attained scales simply with the mean thick-line velocity separation as $\eta \approx v_\infty/\Delta v$ (figure 4(*a*)).

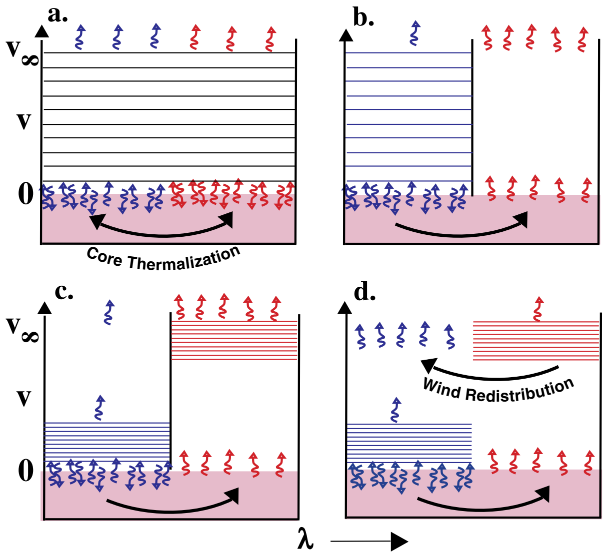

Figure 4. Schematic diagram illustrating the role of line gaps, line bunches and photon thermalization for WR wind momentum deposition. The horizontal lines represent the velocity–frequency spacing of optically thick lines in the wind. The four parts represent (*a*) an effectively gray model, (*b*) a wind with fixed ionization and extensive gaps, (*c*) a wind with ionization stratification that fills gaps and (*d*) the importance of limiting energy redistribution in the wind.

However, realistic line lists are not gray, and leakage through gaps in the line spectral distribution tends to limit the effective scattering to $\eta \lesssim 1$ (figure 4(*b*)). However, the optically thick nature of the WR winds gives them a substantial ionization stratification, and this helps to spread line bunches and so fill in gaps, allowing for more effective global trapping of radiation that makes $\eta > 1$ (figure 4(*c*)). Redistribution of photon energy can reduce the local effectiveness of line driving (figure 4(*d*)). In particular, redistribution by photon thermalization near the stellar core makes it especially difficult for radiation alone to initiate the wind. The relative complexity of WR wind initiation may be associated with the extensive turbulent structure inferred from observed variability in WR wind emission lines. Overall, the understanding of WR winds is perhaps best viewed as an 'opacity problem', i.e. identifying the enhanced opacity that can adequately block the radiation flux throughout the wind and thus drive a WR mass loss that is much greater than from OB stars of comparable luminosity.

Wind instability and variability

There is extensive evidence that hot-star winds are not the smooth, steady outflows idealized in the simple

CAK theory but rather have extensive structure and variability on a variety of scales. The most direct example is the explicit variability often detected in unsaturated absorption troughs of P-Cygni line profiles. To produce such explicit line profile variability, the associated wind structure must be relatively large scale, covering a substantial fraction of the stellar disk. (By comparison, the emission component of P-Cygni profiles is generally much less variable, since it forms from a more global average of the wind.)

Much of the variability can be characterized as 'discrete absorption components'. These begin as broad absorption enhancements in the inner part of the blue absorption trough of an unsaturated P-Cygni line, which then gradually narrow as they drift, over a period of days, toward the blue edge of the profile. Their occurrence is generally irregular, and their apparent acceleration is quite slow, much slower than expected for an element of the steady wind. They may reflect the slow acceleration of mass ejected by some kind of semirandom disturbance from the underlying stellar surface. The exact nature of this is unknown, but might possibly be linked to either magnetic activity or stellar pulsation.

In a second class of explicit variation are the 'periodic absorption modulations'. These occur at regular, sometimes multiple periods, and include both a modulated increase and decrease in the absorption strength. The periods are generally of order a few days, which is consistent with a low-order multiple of the stellar rotation period. They probably represent rotational modulation of wind structure by an unknown kind of surface irregularity (again perhaps magnetic).

The optical emission lines formed in WR winds also commonly show a low- amplitude variability. Such optical lines have been monitored using ground-based telescopes at much higher signal-to-noise ratio than has been typical for the ultraviolet wind lines monitored from OB stars by orbiting satellites. Analysis suggests a turbulent hierarchy of structure extending down to quite small spatial scales.

There is also indirect evidence that OB winds have an extensive, small-scale, turbulent structure. Saturated P-Cygni lines have extended black troughs thought to be a signature that the wind velocity is highly nonmonotonic. Such stars commonly show soft x-ray emission and sometimes also nonthermal radio emission, both of which are thought to originate from embedded wind shocks.

A promising explanation for this small-scale, turbulent structure is the intrinsically strong instability of line driving to small-scale velocity perturbations. As noted above, there is a strong hidden potential in line scattering to drive wind material with accelerations that greatly exceed the mean outward acceleration. Numerical simulations suggest this can indeed lead naturally to a highly structured flow dominated by multiple shock compressions. However, it is not yet clear whether the simulated flow properties can robustly explain key observational features such as the soft x-ray emission.

Frontiers: colliding winds, disks, etc

The theory for line-driven winds is mature enough that it is now finding application in many more complex circumstances. Let us thus briefly summarize some of these frontiers.

One example is colliding winds of close, massive star binaries. In several cases these involve WR + O stars with separations of only a few O-star radii. Generally the WR wind is so much stronger that a simple hydrodynamic ram balance between the two stars is not possible. This suggests the WR wind should simply overwhelm its companion's outflow and penetrate down to the O-star surface. Orbital phase monitoring of such systems suggests, however, that this does not occur, but that a wind–wind interface is generally kept away from the O-star surface. An attractive explanation for this comes from considering the additional pressure provided by the O-star light in a *radiative braking* of the impacting WR wind. Such a radiative deceleration has many of the same properties of the usual wind acceleration, including a crucial role for the Doppler shift of line resonance in allowing a strong line force. An interesting result is that radiative braking is generally only effective if the interaction of O-star light with WR material is characterized by a stronger line opacity than needed for driving the O-star wind. This represents a new twist to the usual 'opacity problem' of how WR stars drive such strong winds and even suggests that observations of WR + O binaries could help to constrain answers to this fundamental question.

Hot stars generally have a rapid rotation, but among the fastest rotators are the BE STARS, which are characterized by strong Balmer line emission thought to originate in a circumstellar disk. A quite appealing idea is that these form naturally as 'wind-compressed disks' by rotational focusing of the star's radiatively driven stellar wind toward the equatorial plane. Initial dynamical simulations that assumed a simple, radial form for the line force generally supported this idea. Surprisingly, however, subsequent, more realistic models show that *nonradial* components of this line force can actually completely *inhibit* the formation of such a disk by effectively driving material away from the equatorial plane. Further analysis has shown how this, and other peculiar properties of line driving, follows naturally from the characteristic scaling of the line force with the velocity gradient.

Finally, line-driven winds can also originate from luminous accretion disks, such as those found in cataclysmic variables (see also CATACLYSMIC BINARIES: CLASSICAL AND RECURRENT NOVAE). Recent work has thus focused on extending the basic CAK theory to account for key differences from the stellar case, for example the 2D nature of the flow and the peculiar nonmonotonic variation of the effective gravity, which initially increases from the disk plane before eventually falling off with distance. There is even renewed interest in developing such disk wind models to explain the broad-line flows observed from ACTIVE GALACTIC NUCLEI and QUASARS.

Bibliography

The theory of hot-star winds is nearly three decades old, and in this time there has amassed an appropriately large literature.

The basic notion of line driving was first broached in a prescient paper by

Milne E A 1926 *Mon. Not. R. Astron. Soc.* **86** 459

The seminal work laying out the CAK formalism still used today was in

Abbott D 1982 *Astrophys. J.* **263** 723

Castor J I, Abbott D C and Klein R I 1975 *Astrophys. J.* **195** 157

The key Sobolev approximation was developed in

Sobolev V V 1960 *Moving Envelopes of Stars* (Cambridge, MA: Harvard University Press)

The reformulation of the CAK scalings in terms of the Q of line resonances was by

Gayley K G *Astrophys. J.* **454** 410

A key paper testing the validity of the Sobolev approach was also one of the first to relax the CAK point-star approximation:

Pauldrach A, Puls J and Kudritzki R 1986 *Astron. Astrophys.* **164** 86

Wind variability is the subject of several recent conference proceedings:

Kaper L and Fullerton A (eds) 1998 *Cyclical Variability in Stellar Winds* (Dordrecht: Kluwer)

Moffat A, Owocki S, St-Louis N and Fullerton A (eds) 1994 *Instability and Variability in Hot Star Winds* (Dordrecht: Kluwer)

Wolf B, Fullerton A and Stahl O (eds) 1999 *Variable and Non-spherical Stellar Winds in Luminous Hot Stars (Springer Lecture Series in Physics)*

A recent paper on instability, which also addresses the general validity of the Sobolev approach, is

Owocki S P and Puls J 1999 *Astrophys. J.* **510** 355

WR stars have also been the subject of several recent conferences, with proceedings containing several reviews on the dynamics of WR winds both for single stars and in binary systems:

van der Hucht K, Koenigsberger G and Eenens P (eds) 1999 *Wolf–Rayet Phenomena in Massive Stars and Starburst Galaxies (IAU Symposium 143)* (Dordrecht: Kluwer)

Some general reviews on hot-star winds can be found in

Cassinelli J 1979 *Ann. Rev. Astron. Astrophys.* **17** 275

Howarth I (ed) 1997 *Boulder–Munich Workshop II* (San Francisco: Astronomical Society of the Pacific)

Kudritzki R and Hummer D 1990 *Ann. Rev. Astron. Astrophys.* **28** 303

Owocki S 1990 *Rev. Mod. Astron.* **3** 98

Puls J *et al* 1993 *Rev. Mod. Astron.* **6** 271

Finally, an essential general reference is the new introductory book on stellar winds:

Lamers H J G L M and Cassinelli J 1999 *Introduction to Stellar Winds* (Cambridge: Cambridge University Press)

Stanley Owocki