Phys 811 Questionaire

Here are quick answers to the questionaire. I had in mind that 1,4,8 were questions that you should have been able to give a reasonable answer. 2&5 you might have had some proir knowledge to give a partial answer. 3,6,7 are really part of the material for 811. I wrote this up partly to have something on the course web site for you to test access ability, but also so you might better assess if this is the right course for you.

■ Physics

■ 1. Describe the hydrogen atom energy spectrum, including just the Coulomb potential.

The energy levels of the hydrogen atom are given by $E_n = \frac{\alpha^2}{2n^2}\mu_e = \frac{13.6}{n^2}$ eV, where n is the principle quantum number, α is the fine structure constant, and μ_e is the reduced mass for the two body system.

■ 2. Give the degeneracy for the first 3 energy levels.

The simple hydrogen atom energy level depends only on the principle quantum numbers, but not on angular momentum or spin. The allowed values of angular momentum are $l < n$, so the ground state has only $l = 0$. Because of the electron spin, there are 2 degenerate $1s$ ground states. For $n = 2$, both $l = 0, 1$ are allowed. For each value of l, there are $2l + 1$degenerate states, so the total number of states is $N_2 = 2(1+3) = 8$. These are the $1p$ and $2s$ states. Similarly, for $n = 3$, the number of degenerate states is $N_3 = 2(1+3+5)$, which have atomic labels of $1d$, $2p$ and $3s$.

■ 3. Write down the generators of SU(2). (These are also known as Pauli matricies.)

By convention, $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

■ 4. Write the commutation relations for the generators of angular momentum, J_x, J_y, J_z, J^2

$[J^2, J_i] = 0$, $[J_i, J_j] = i\hbar\epsilon_{ijk} J_k$

■ 5. Write the hamiltonian for a non-relativistic charged particle in an electro-magnetic field.

$H = \frac{(p-eA)^2}{2m} - e\phi$, where ϕ is the Coulomb potential and A is the vector potential.

■ 6. Give an expression for the translation operator $T(x)$ in terms of the momentum p.

The momentum operator $p = -i\hbar\frac{\partial}{\partial x}$ is the generator of translations $T(x) = e^{-\frac{i}{\hbar}xp}$.

■ 7. What is Fermi's Golden rule?

Fermi's Golden Rule is a formula for the transition rate from an initial state $|i\rangle$ to a final state $|f\rangle$ under the influence of a harmonic perturbation $V = V_0 e^{i\omega t}$, calculated to first order in perturbation theory.

$$\Gamma = \frac{2\pi}{\hbar} |\langle f \mid V_0 \mid i \rangle|^2 \rho(E_f)$$

where $E_f = E_i + \omega$ is the final state energy, and $\rho(E_f)$ is the density of states at E_f.

- **8. A particle of mass m is in a state described by a wave function $\Psi(x)$. Give an expression for expectation of the velocity of the particle**

$\langle v \rangle = \langle \frac{p}{m} \rangle = -\frac{i\hbar}{m} \frac{\int \Psi^*(x) \frac{\partial}{\partial x} \Psi(x)}{\int \Psi^*(x) \Psi(x)}$, where the denominator allows for the possibility that the waveform is not normalized.