5. **Blowing up a planet**

In the original *Star Wars* movie, a “Death Ray” is used to blow up the planet Alderan in just a few (say 10) seconds.

(a) Assuming Alderan has the same mass $M_e$ and radius $R_e$ as our Earth, estimate its total gravitational binding energy $U$ in Joules (J), under the assumption that it has roughly a constant mass density $\rho$ throughout.

$$BE = \frac{3}{5}GM_eR_e^2 = 2.2 \times 10^{32} J$$  \hspace{1cm} (7.1)

(b) Estimate then the minimum power $P$ of the Death Ray, giving your answer in both Watts ($W=J/s$), and in solar luminosities, $P/L_\odot$?

$$P = \frac{BE}{t} = 2.2 \times 10^{31} W = 57,500L_\odot$$  \hspace{1cm} (7.2)

(c) Assuming the Death Ray originates from a circular aperture of radius $R = 1$ km, compute its initial energy flux $F_a$ in $W/m^2$.

$$F_a = \frac{P}{A} = \frac{2.2 \times 10^{31} W}{\pi(10^6 m)^2} = 7.1 \times 10^{24} W/m^2$$  \hspace{1cm} (7.3)

(d) What is the ratio of this to the surface flux of the Sun, $F_a/F_\odot$?

$$\frac{F_a}{F_\odot} = \frac{F_a}{(L_\odot/4\pi R_\odot^2)} = 1.1 \times 10^{17}$$  \hspace{1cm} (7.4)

(e) What would the temperature $T$ of the Death Ray have to be (in K) if this flux were emitted by blackbody radiation? (Hint: recall the surface temperature of the Sun is about $T_\odot \approx 6000$ K.)

$$T = T_\odot \left( \frac{F_a}{F_\odot} \right)^{1/4} = 1.1 \times 10^8 \text{ K}$$  \hspace{1cm} (7.5)

(f) Assuming the focus of the Death Ray make this full power just cover uniformly the projected area of Alderan, compute the associated surface energy flux $F_a$ in $W/m^2$ on that planet.

$$F_a = \frac{P}{\pi R_e^2} = \frac{2.2 \times 10^{31} W}{\pi(6.4 \times 10^6 m)^2} = 1.7 \times 10^{17} W/m^2$$  \hspace{1cm} (7.6)

(g) What is the ratio of this to the surface flux of the Sun, $F_a/F_\odot$?

$$\frac{F_a}{F_\odot} = 6.6 \times 10^8$$  \hspace{1cm} (7.7)
3. Stellar properties and lifetime

A Main-Sequence star with parallax \( p = 0.01 \) arcsec has an apparent magnitude \( m = +2.5 \).

(a) What is its distance \( d \), in pc?

\[
\frac{d}{\text{pc}} = \frac{\text{arcsec}}{p} = 100 \Rightarrow d = 100\text{pc} \quad (8.1)
\]

(b) What is its luminosity \( L \), in \( L_\odot \)?

\[
m = +2.5 = 5 - 2.5 \log(L/L_\odot) + 5 \log(d/10\text{pc}) = 5 - 2.5 \log(L/L_\odot) + 5
\]

\[
\Rightarrow L = 10^3 L_\odot \quad (8.2)
\]

(c) What is its mass \( M \), in \( M_\odot \)?

For main-sequence star

\[
\frac{L}{L_\odot} = \left( \frac{M}{M_\odot} \right)^3 \Rightarrow M = 10M_\odot \quad (8.3)
\]

(d) What is its main sequence lifetime \( t_{\text{ms}} \), in Myr?

For main-sequence star

\[
t_{\text{ms}} = 10\text{Gyr} \left( \frac{M}{M_\odot} \right)^{-2} = 100\text{Myr} \quad (8.4)
\]

(e) What is its estimated radius \( R \), in \( R_\odot \)?

On main sequence, \( R/R_\odot \approx M/M_\odot \), so \( R \approx 10R_\odot \).

(f) What is its associated surface temperature \( T \), in K?

\[
10^3 = 10^2 \left( \frac{T}{T_\odot} \right)^4 \Rightarrow T = 6000 K \ 10^{1/4} = 10,700 K. \quad (8.5)
\]

(g) Suppose this star is in a cluster and sits right below the main-sequence turnoff point in its H-R diagram. What is the age of the cluster, \( t_{\text{cluster}} \), in Myr?

\[
t_{\text{age}} = t_{\text{ms}} = 10\text{Gyr} \left( \frac{M}{M_\odot} \right)^{-2} = 100\text{Myr} \quad (8.6)
\]
4. Absorption by coal dust.

Imagine you’re a coal miner working under a bright, 1400-watt lamp that is at a distance of 10 meters away from your work location.

(a) Assuming the lamp emits its light isotropically, what is the flux of light on your workspace, in watt/m²? (Assume the coal mine walls are perfect absorbers, i.e with zero albedo). How does this compare with the flux of sunlight on a sunny day at the surface?

\[
F = \frac{L}{4\pi d^2} = 1400 \text{W}/(4\pi 10^2 \text{m}^2) = 1.2 \text{W/m}^2
\]  
(12.2)

By comparison, \(F_\odot = 1400 \text{W/m}^2\) or fraction \(F/F_\odot = 8.6 \times 10^{-4}\).

(b) Now suppose there is a cave-in that fills the mine with black, spherical coal dust particles of diameter 0.1 mm, and with a uniform number density of 20 particles per cubic centimeter. What is the mean-free-path \(\ell\) (in m) of light in the mine?

\[
\ell = \frac{1}{n \pi D^2/4} = 6.4 \text{m}
\]  
(12.3)

(c) What is the optical depth \(\tau\) between you and the lamp?

\[
\tau = \frac{d}{\ell} = 1.6
\]  
(12.4)

(d) What is now the flux on your workspace?

\[
F = F_\odot e^{-\tau} = 1.2 \text{W/m}^2 e^{-1.6} = 0.24 \text{W/m}^2
\]  
(12.5)

(e) What distance had this flux value before the cave in?

\[
d = \sqrt{L/4\pi F} = 22 \text{m}
\]  
(12.6)

(f) How close would you have to move the lamp to make the flux on your workspace be the same as before the cave-in?

\[
\frac{L_o}{4\pi d^2} e^{-d/\ell} = \frac{L_o}{4\pi d_0^2} \Rightarrow de^{d/2\ell} = d_o
\]  
(12.7)

Can solve this by iteration, or use:

\[
d = 2\ell \text{ProductLog}(d_0/2\ell) = 6.2 \text{m}
\]  
(12.8)
5. Absorption by interstellar dust. Imagine you’re an astronomer observing a star with luminosity $1000L_\odot$ that is at a distance of 10 parsec.

(a) What is the flux of light you observe? Give your answer in both $W/m^2$ and $L_\odot/\text{parsec}^2$.

$$F = \frac{L}{4\pi d^2} = 0.8L_\odot/\text{pc}^2 = 3 \times 10^{-7} W/m^2 \quad (12.8)$$

(b) Now assume the space between you and the star contains spherical dust particles of diameter 1 micron and number density of 6000 particles per cubic kilometer. Assuming the dust absorbs or scatters light in proportion to its geometric cross-section, what is the optical depth between you and the star?

$$\sigma = \frac{\pi}{4} D^2 = 8 \times 10^{-13} m^2; \quad n = \frac{6000}{10^9 m^3} = 6 \times 10^{-6} m^{-3}$$

$$\tau = n\sigma d = 6 \times 10^{-6} \times 8 \times 10^{-13} \times 3 \times 10^{17} = 1.4 \quad (12.9)$$

(c) What is now the flux you observe? Again give your answer in both watt/m$^2$ and $L_\odot/\text{parsec}^2$.

$$F = \frac{L}{4\pi d^2} e^{-\tau} = 0.2L_\odot/\text{pc}^2 = 7 \times 10^{-8} W/m^2 \quad (12.10)$$

(d) If you knew the star’s luminosity but didn’t know about the dust, what distance (in pc) would you infer for the star based on your observed flux?

$$d = \sqrt{\frac{L}{4\pi F_{\text{obs}}}} = \sqrt{1000/(4\pi 0.2)} \text{pc} = 20\text{pc} \quad (12.11)$$

(e) What is the change in the apparent magnitude of this star resulting from the dust absorption.

$$A = \Delta m = m_{\text{obs}} - m = 1.09 \tau = 1.5 \quad (12.12)$$
4. Scale height in Earth’s atmosphere

(a) The Earth’s atmosphere is mostly diatomic nitrogen, with molecular weight \( \mu \approx 28m_p \). For a typical temperature on a spring day (~50°F), compute the isothermal sound speed, \( c_s \), in km/s, and as a ratio to the orbital speed in low-Earth-orbit, \( v_{orb} = 7.9 \text{ km/s} \).

\[
c_s = \sqrt{\frac{kT}{\mu}} = \sqrt{\frac{1.38 \times 10^{-16} \text{ 283}}{28 \times 1.67 \times 10^{-24}}} = 290 \text{ m/s} = 0.037 v_{orb}
\] (15.1)

(b) Use this and Earth’s surface gravity to compute the atmospheric scale height \( H \) for the Earth (in km), and its ratio to Earth’s radius, \( H/R_e \). How does the latter compare with \( c_s/v_{orb} \)?

\[
H = \frac{c_s^2}{g} = \frac{2.9^2 \times 10^4}{9.8} = 8.6 \text{ km}
\] (15.2)

\[
\frac{H}{R_e} = 0.0014 = 0.037^2
\] (15.3)

which is thus the square of \( c_s/v_{orb} \).

(c) The pressure at sea level is defined as 1 atmosphere (atm). Ignoring any temperature change of the atmosphere, estimate the pressure (in atm) at a typical altitude \( h = 300 \text{ km} \) for an orbiting satellite.

\[
\frac{P_{orb}}{P_{sea}} = e^{-z/H} = e^{-300/8.6} = 7 \times 10^{-16}
\] (15.4)

(d) A satellite in circular orbit at this altitude of \( h = 300 \text{ km} \) will typically stay in orbit for about decade. If the temperature of the remaining gas at this height is twice that of the Earth’s surface, estimate how much higher the orbital height would have to be to double this orbital lifetime.

Assume orbital lifetime is proportional to inverse of density, \( 1/\rho \). Then since \( H \sim T \), with gravity nearly the same as at the surface, we have

\[
\frac{\rho_z}{\rho_{300}} = e^{-\Delta z/2H} = 1/2 \Rightarrow \Delta z = 2H \ln(2) = 12 \text{ km}
\] (15.5)