Instability & Mass Loss near the Eddington Limit

S. P. Owocki & N. J. Shaviv

Abstract We review the physics of continuum-driven mass loss and the role it plays in η Carinae and other LBVs. Unlike line-driven mass loss, which is inherently limited by self-shadowing, continuum driving can in principle lead to mass-loss rates up to the "photon-tiring" limit, for which the energy to lift the outflow becomes equal to the stellar luminosity. We discuss how luminous atmospheres give rise to a clumped atmosphere, and how the associated "porosity" can regulate continuum-driven mass loss. We also summarize recent time-dependent simulations of how a base mass flux that exceeds the tiring limit can lead to flow stagnation and a complex, time-dependent combination of inflow and outflow regions. A general result is thus that porosity-regulated continuum driving in super-Eddington epochs can explain the large, near tiring-limit mass loss inferred for LBV giant eruptions. However, while these extreme mass loss rates are allowed for dynamically long periods, they cannot be sustained for an evolutionary timescale, and so ultimately it is stellar structure and evolution that sets the overall level the mass loss.

Key words: stars: early-type, stars: winds, outflows, stars: mass loss, stars: activity

1 Introduction

The LBV giant eruptions that can pose as "supernova imposters" are characterized by both a strongly enhanced brightness and a substantial ejection of mass. A particularly extreme example is η Carinae, which during the giant eruption of 1840-60 is

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estimated to have ejected some $10 M_\odot$ at moderately high speeds of 500-800 km/s [31], implying a wind kinetic energy loss rate that rivals the extreme radiative luminosity $L \approx 20 \times 10^6 L_\odot$ estimated for this epoch. The associated total energy is only about 1% of the 10^{51} erg for a typical supernova, but given that the star remains intact, such eruptions seem difficult to understand in terms of the singular explosions of core-collapse supernovae or alike. Moreover, such extreme mass loss is orders of magnitude greater than can be readily explained by the standard CAK [6], steady-state wind model for radiative driving by scattering in a ensemble of lines from metal ions.

Instead, the observational association of η Carinae and other LBVs as being near an apparent upper limit in luminosity for observed stars [15, 16] has led to the general view that such strong episodes of mass loss may stem from the star approaching or exceeding the Eddington limit, at which even the continuum force associated with just electron scattering exceeds the inward force of gravity. Since the continuum force can penetrate into the deep interior, approaching the Eddington limits leads to a dominance of radiation pressure over gas pressure in supporting the stellar envelope against gravity. In standard, 1-D equilibrium models, the structural adjustment keeps the star sub-Eddington. But various kinds of instabilites in the deep interior or envelope can disrupt this equilibrium, with an energy release that induces an interval of super-Eddington luminosity.

This can lead to a situation analogous to that in classical novae, wherein the sudden onset of shell burning induces a super-Eddington brightness that last for several months (e.g., Nova LMC 1988 #1, which was super-Eddington for about 50 days [25]). Since this is much longer than any dynamical timescale in the nova system, it can result in a quasi-steady wind mass loss, driven however by continuum rather than line opacity. In fact, it was realized that optically thick continuum-driven winds explain many characteristics of these objects long before it was understood how super-Eddington luminosities could arise [2]. Note also that, unlike LBVs with poorly determined masses, novae have the Chandrasekhar mass as a strict upper limit, thus it is unequivocal that the objects are indeed super-Eddington, and their continuum-driven winds are indeed accelerated under such conditions [29].

Whatever mechanism may trigger such a super-Eddington brightening in the LBVs, a key general issue is how the deeply penetrating effect of continuum driving is regulated to keep the stellar interior gravitationally bound, while allowing initiation of a sustainable mass loss from near the surface.

As discussed below, it seems that interior convection, flow stagnation, and the ‘porosity’ of a clumped medium may all play a role. A likely net result is a mass loss rate that approaches the “photon-tiring limit”, associated with the finite energy available to lift material out of the star’s gravitational potential. This is several orders of magnitude higher than can be achieved by line-driving, and is indeed comparable to the mass loss rate, $\sim 1 M_\odot\,\mathrm{yr}^{-1}$, inferred for the giant eruption of η Carinae [22].

However, the associated mass loss time scale, $t_{\dot{M}} \equiv M/\dot{M} \sim 100$ yr, is so short that this state can only be sustained for a few years before inducing fundamental readjustments in the stellar structure. Once that quenches the super-Eddington condition, it may take a much longer, thermal relaxation time to initiate a new eruption.

But even with such a small "duty cycle", the net effect can dramatically reduce the stellar mass over an evolutionary timescale. Thus, such LBV mass loss may play a central role in the evolution and final fate of the most massive stars, and indeed may be a key factor in setting the stellar upper mass limit. Moreover, unlike the case for line-driving from metal ions, these continuum-driven processes are not (directly) dependent on metalicity, and so may even play a similar role in the formation, evolution, and fate of the first generation of massive stars, which are thought be key to reionizing the universe following the recombination epoch of the Big Bang.

The discussion below elaborates further on this role of radiation forces for both the interior structure (§2) and mass loss (§3) of massive stars. A concluding discussion (§4) considers general issues related to the energy source of the giant eruptions and various implications for massive star evolution near the Eddington limit.

2 Radiation Pressure in Massive-Star Envelopes

The key to understanding the internal structure of massive stars is to recognize the different effects that arise in the presence of very strong radiative fields. In §2.1, we begin by discussing the radiative force and the Eddington limit. In §2.2, we demonstrate how the radiative pressure changes the mass luminosity relation. In §2.3, we show that as the Eddington luminosity is approached, convection is necessarily excited, implying that the interior of a star never becomes super-Eddington. In §2.4 we discuss the instabilities that porosify the outer atmosphere. The combined effects allow us to build a coherent picture, in §2.5, for the structure of the *static* parts of super-Eddington stars, such as η-Car during its giant eruption.

2.1 Radiative force and the Eddington limit

As a basis for understanding radiative driving, let us begin by considering the general form for the radiative acceleration $\mathbf{g}_{rad}$ associated with a specific opacity κ_ν (a.k.a. the mass absorption coefficient, with CGS units cm^2/g) for interaction of stellar material with a radiative flux $\mathbf{F}_\nu$ at photon frequency ν,

$$\mathbf{g}_{rad} = \int_0^\infty d\nu\, \kappa_\nu \mathbf{F}_\nu / c\,, \tag{1}$$

with c the speed of light. In general the opacity κ_ν includes both broad-band continuum processes – e.g. Thomson scattering of electrons, and bound-free or free-free absorption – and bound-bound transitions associated with line absorption and/or scattering.

In the static envelope and atmosphere, the reduction in flux in saturated lines keeps the associated line-force small, and so to a good approximation the overall radiative acceleration is dominated by a continuum contribution characterized by electron scattering. Because such opacity is gray (frequency-independent), it can be pulled outside the frequency integration in eq. (1). In an idealized, spherically symmetric, radiative envelope, the bolometric flux $F \equiv \int_0^\infty d\nu F_\nu$ is thus purely radial, and at any local radius r is simply set by $F = L/4\pi r^2$, where L is the total bolometric luminosity generated in the stellar core. The radiative acceleration associated with such a gray opacity κ is thus given by

$$g_{rad} = \frac{\kappa F}{c} = \frac{\kappa L}{4\pi r^2 c} \equiv \Gamma g\,, \tag{2}$$

where the last equality introduces the Eddington parameter, $\Gamma \equiv \kappa L/4\pi GMc$, which gives the ratio of the radiative acceleration to the local gravitational acceleration, $g \equiv GM/r^2$, with G the gravitation constant and M the stellar mass.

A key point is that, since both gravity and radiative flux have the same r^{-2} decline with radius r, this ratio can often be considered nearly independent of radius, that is when the opacity κ, radiative luminosity L, and mass M are all fixed.[1] For the classical case of pure electron scattering, the Eddington parameter scales as

$$\Gamma_e = 2.6 \times 10^{-5} \frac{L}{L_\odot} \frac{M_\odot}{M} \tag{3}$$

As discussed in the next subsection, stellar luminosity generally scales with a high power of the stellar mass, i.e. $L \propto M^3$, and so massive stars with $M > 10 M_\odot$ generally have electron Eddington parameters of order $\Gamma_e \approx 0.1 - 1$. Indeed, $\Gamma_e \equiv 1$ defines the *Eddington limit*, for which the entire star would become unbound, at least in this idealized model of a 1-D, spherically symmetric, radiative envelope.

It should thus be emphasized, however, that this does not represent an appropriate condition for the steady-state mass loss characteristic of a stellar wind, since that requires an outwardly increasing radiative force that goes from being *less* than gravity in a bound stellar envelope to *exceeding* gravity in the outflowing stellar wind. §3 summarizes how the necessary force modulation can occur through line-desaturation for line driving, and through porosity of spatial structure for continuum driving.

2.2 Stellar structure scaling for luminosity vs. mass

The structure of a stellar envelope is set by the dual requirements for momentum balance and energy transport. The former is described through the equation for hydrostatic equilibrium, modified to account for a factor $1 - \Gamma$ reduction in the effective gravity, due to the radiation force.

$$-\frac{1}{\rho}\frac{dP}{dr} = \frac{GM(1-\Gamma)}{r^2}\,. \tag{4}$$

[1] As discussed below, there are various circumstances in which this is not the case.

Using the ideal gas law for pressure $P \sim \rho T$, a single point evaluation with mass M and radius R replacing density $\rho \sim M/R^3$ implies a characteristic interior temperature that scales as

$$T \sim \frac{M(1-\Gamma)}{R} \,. \tag{5}$$

Through most of the stellar envelope, the energy flux $F = L/4\pi r^2$ is tranported by diffusion of radiative energy density $U_{rad} \sim T^4$,

$$F = -\frac{1}{\kappa\rho c}\frac{dU_{rad}}{dr} \,, \tag{6}$$

which implies the dimensional scaling

$$L \sim \frac{R^4 T^4}{M} \,. \tag{7}$$

When combined with eq. (5) for the interior temperature, we see that the *radius cancels* in the scaling of luminosity, yielding

$$L \sim M^3 \, (1-\Gamma)^4 \,. \tag{8}$$

Quite remarkably, this scaling does not depend explicitly on the nature of energy generation in the stellar core, but is strictly a property of the envelope structure[2].

Figure 2.2 shows a log-log plot of the resulting variation of luminosity vs. mass. For low-mass stars, it implies a strong $L \sim M^3$ scaling, but as this forces stars to approach the Eddington limit, the $1-\Gamma$ term acts as a strong repeller away from that limit, causing a broad bend toward a linear asymptotic scaling, $L \sim M$.

Formally, this scaling suggests it is in principle possible to have stars with arbitrarily large mass, approaching arbitrarily close to the Eddington limit. But surveys of dense young clusters are providing increasingly strong evidence for a sharp cutoff in the stellar mass distribution at about $M \approx 150-200\ M_\odot$ [41, 8, 20].

Note that this inferred upper mass limit corresponds closely to the center of the bend region in figure 2.2. This is just somewhat beyond the transition, at $\Gamma \approx 1/2$, to where radiation plays the dominant role in supporting the star against gravity, implying a radiation pressure that is greater than gas pressure, $P_{rad} > P_{gas}$. Somewhat analagous to having a heavier fluid support a lighter one, such a configuration may be subject to various kinds on intrinsic instabilities, leading to spatial clumping and/or the brightness variations that trigger LBV eruptions [37, 26, 27, 28]. The large associated LBV mass loss of such near-Eddington-limit stars thus could play a key role in setting the stellar upper mass limit.

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[2] Of course, this simple one-point scaling relation does have to be modified to accomodate gradients in the molecular weight as a star evolves from the zero-age main sequence, and it breaks down altogether in the coolest stars (both giants and dwarfs), for which convection dominates the envelope energy transport.

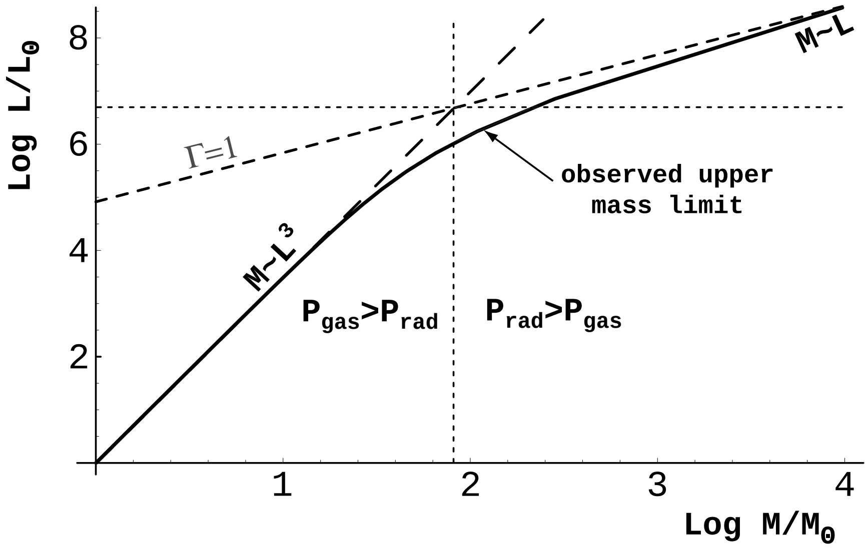

Fig. 1 Log-log plot of the scaling of stellar luminosity L vs. mass M implied by eq. (8).

2.3 *Convective instability of a super-Eddington stellar snterior*

It should first be emphasized that locally exceeding the Eddington limit need *not* necessarily lead to initiation of a mass outflow. As first shown by Joss et al. [17], in a stellar envelope allowing the Eddington parameter $\Gamma \rightarrow 1$ generally implies through the Schwarzschild criterion that material becomes *convectively unstable*. Since convection in such deep layers is highly efficient at transporting the energy, the contribution from the radiative flux is reduced, thereby lowering the associated radiative Eddington factor away from unity.

This suggests that a radiatively driven outflow should only be initiated *outside* the region where convection is *efficient*. An upper bound to the convective energy flux is set by

$$F_{conv} \approx v_{conv}\, l\, dU/dr \lesssim a\, H\, dP/dr \approx a^3 \rho \,, \tag{9}$$

where v_{conv}, l, and U are the convective velocity, mixing length, and internal energy density, and a, H, P, and ρ are the sound speed, pressure scale height, pressure, and mass density. Setting this maximum convective flux equal to the total stellar energy flux $L/4\pi r^2$ yields an estimate for the maximum mass loss rate that can be initiated by radiative driving,

$$\dot{M} \leq \frac{L}{a^2} \equiv \dot{M}_{max,conv} = \frac{v_{esc}^2}{2a^2} \dot{M}_{tir} \,, \tag{10}$$

where the last equality emphasizes that, for the usual case of a sound speed much smaller than the local escape speed, $a \ll v_{esc}$, such a mass loss would generally be well in excess of the photon-tiring limit set by the energy available to lift the material out of the star's gravitational potential, given by

$$\dot{M}_{tir} = \frac{L}{v_{esc}^2/2} = \frac{L}{GM/R} = 0.032\,\frac{M_\odot}{\mathrm{yr}}\;\frac{L}{10^6 L_\odot}\,\frac{M_\odot/R_\odot}{M/R}\,. \tag{11}$$

In other words, if a wind were to originate from where convection becomes inefficient, the mass loss would be so large that it would use all the available luminosity to accelerate out of the gravitational potential, implying that any such outflow would necessarily stagnate at some finite radius. In fact, no consistent super-Eddington solution can be obtained with a sonic point located at the top of the convection layer [27]. Thus, if we are to explain the super-Eddington episode in η-Car and other objects, we need to shift the critical point, where the net force vanishes, higher up the atmosphere to regions with a much lower density.

2.4 Radiative instability and porosification of atmospheres

Since the critical point in known super-Eddington systems resides at relatively low densities, these stars must necessarily have a layer above where convection ends, which is super-Eddington in terms of the luminosity, and yet *effectively* sub-Eddington in terms of the radiative force. The key to understanding the existence of this layer includes two separate aspects. First, as an atmosphere approaches the Eddington luminosity, one of many possible instabilities is triggered and gives rise to inhomogeneities. Second, the introduction of optically thick inhomogeneities (e.g., clumps), reduces the effective opacity, thus allowing for larger fluxes without increasing the radiative force.

Dating back to early work by Spiegel [35, 36], there have been speculations that atmospheres supported by radiation pressure would likely exhibit instabilities not unlike that of Rayleigh-Taylor, associated with the support of a heavy fluid by a lighter one, leading to formation of "photon bubbles".

More-recent, quantitative stability analyses [37, 28] do lead to the conclusion that even a simple case of a pure "Thomson atmosphere" —i.e., supported by Thomson scattering of radiation by free electrons—would be subject to intrinsic instabilities for development of lateral inhomogeneities. The analysis by Shaviv [28] suggests in particular that these instabilities share many similar properties to the excitation of strange mode pulsations [11, 23]. For example, they are favored when radiation pressure dominates over gas pressure. Moreover, both arise when the temperature perturbation term in the effective equation of state for the gas becomes non-local. In strange mode instabilities, the term arises because the temperature in the diffusion limit depends on the radial gradient of the opacity perturbations. In the lateral instability, the term depends on the lateral radiative flux, which arises from non-radial structure on a scale of the vertical scale height. When conditions of a pure Thomson atmosphere are alleviated, more instabilities exist. There are of course the aforementioned strange mode instabilities which require a non-Thomson opacity. But if magnetic fields are introduced, even more instabilities come into play [1, 9, 3, 4].

We stress however that the specific physical origin of the instabilities is not fundamental to our discussion here. The essential point is that as atmospheres approach the Eddington limit, non-radial instabilities do exist to make the atmospheres inhomogeneous, while the typical length scale expected is that of the vertical scale height.

Another key point is that, once instabilities are excited in an atmosphere, the unstable modes will grow to become nonlinear. The resulting inhomogeneities play an important role because, once introduced, they change the ratio between the radiative *flux* through the system and the radiative *force*.

In the presence of inhomogeneities, the flux through the system can be written as the volume average of the flux: $F_{\rm avr} = \langle F \rangle_V$. On the other hand, the force exerted by this flux is $f_{\rm avr} = \langle F \kappa_V \rangle_V$, thus, an effective opacity can be defined as [26]:

$$\kappa_{\rm eff} \equiv \frac{f_{\rm avr}}{F_{\rm avr}} = \frac{\left\langle F \kappa_V \right\rangle_V}{\langle F \rangle_V}. \tag{12}$$

The effect is quite analogous to having opacity variations in *frequency* space, i.e., to non-gray atmospheres. In that case, the Rosseland mean is used to calculate the radiative flux through the system, while the force mean of the opacity, which is the flux-weighted opacity, is used for the net driving. So the two cases are similar, with inhomogeneities in either frequency or in real space.

For a few unique opacity laws, the effective opacity can increase. More generally, however, as is the special case of Thomson scattering, the effective opacity is reduced once any inhomogeneities are present.

One example where $\kappa_{\rm eff}$ can be calculated is the limit of small isotropic perturbations in the optically thick limit of a Thomson-scattering atmosphere that has a negligible gas heat capacity, such that $\nabla \cdot F = 0$. This limit corresponds to the top layers of an atmosphere of a luminous object (yet deep enough for the inhomogeneities to remain opaque). In this case [26],

$$\kappa_{\rm eff} = \kappa_0 \left[1 - \left(\frac{d-1}{d} \right) \sigma^2 \right], \tag{13}$$

where σ is the normalized standard deviation of ρ, and d is the dimension of the system. This result demonstrates that inhomogeneities reduce the effective opacity, but the reduction does not take place in a 1-D system. In other words, the porosity effect is intrinsically *non-radial*, and once it arises, it is responsible for a lower *effective* opacity.

Because the "porous" state depends on the non-linear behavior of the relevant instabilities, and in particular, the mechanism saturating them, there are currently no ab initio calculations predicting the characteristics of the nonlinear state, such as $\kappa_{\rm eff}(\Gamma)$. This will have to wait for elaborate radiative hydrodynamic simulations.

2.5 The “static” structure of super-Eddington stars

The effects described in the previous subsections combine to give an overall picture for how stars can surpass the Eddington limit yet remain stable. This complex spatial structure is depicted in figure 2.

(A) As elaborated upon in §2.3, deep inside the star where the density is sufficiently high, any excess flux above the Eddington luminosity is necessarily advected through convection. Thus, we have a bound layer with $L_{\rm rad} < L_{\rm Edd} < L_{\rm tot}$.

(B) At lower densities, where convection is inefficient, radiative instabilities necessarily force the atmosphere to become inhomogeneous. This reduces the effective opacity and thus increases the effective Eddington luminosity $L_{\rm eff}$. In other words, this layer is bound, not because the flux is lowered (as it does in the convective regions), but because the opacity is reduced. That is $L_{\rm Edd} < L_{\rm rad} = L < L_{\rm eff}$ in the layer.

(C) Opacity reduction can operate only as long as the inhomogeneity clumps are optically thick. At lower densities still, the clumps lose their opaqueness and so the effective opacity recovers the microscopic value (and thus $L_{\rm eff}$ to $L_{\rm Edd}$). A sonic/critical point of a wind will therefore be located where $L = L_{\rm eff} \gtrsim L_{\rm Edd}$. More about this in §3.

(D) Since the mass loss rate is large, the wind is optically thick and the photosphere resides in the wind itself, where geometrical dilution makes it transparent enough.

3 Radiatively Driven Mass Loss

3.1 Line-driven stellar winds

As a basis for developing a model for continuum-driven mass loss from super-Eddington phases of LBV stars, let us next review the more well-established theory for steady line-driven winds.

The resonant nature of line (bound-bound) absorption leads to an opacity that is inherently much stronger than from free electrons. For example, in the somewhat idealized, optically thin limit that all the line opacity could be illuminated with a flat, unattenuated continuum spectrum with the full stellar luminosity, the total line-

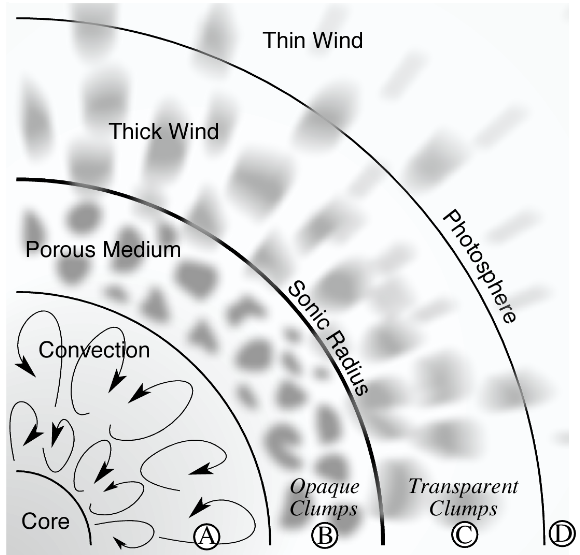

Fig. 2 The structure of a super-Eddington star. The regions are described in the text (§2.5).

force would exceed the free-electron force by a huge factor, of order $Q \approx 2000$ [10]. For massive stars with typical electron Eddington parameters within a factor two of unity, $\Gamma_e \approx 1/2$, this implies a net outward line acceleration that could be as high as $\Gamma_{lines} \approx Q\Gamma_e \approx 1000$ times the acceleration of gravity!

Of course, this does not generally occur in practice because of the self-absorption of the lines. For a single line with frequency-integrated opacity $\kappa_q = q\kappa_e$, the reduction in the optically thin line-acceleration $q\Gamma_e$ can be written as

$$\Gamma_{line} \approx q\Gamma_e \, \frac{1-e^{-qt}}{qt} \,, \tag{14}$$

where $t \equiv \kappa_e \rho c/(dv/dr)$ is the Sobolev optical depth of a line with unit strength, $q = 1$ [34, 6]. Within the standard CAK line-driven wind theory, the number distribution of spectral lines vs. line-strength is approximated as a power law. The associated line-ensemble-integrated radiation force is then reduced by a factor $1/(Qt)^\alpha$ from the optically thin value,

$$\Gamma_{lines} = \frac{Q\Gamma_e}{(1-\alpha)(Qt)^\alpha} \propto \left(\frac{1}{\rho}\,\frac{dv}{dr}\right)^\alpha \,. \tag{15}$$

The latter proportionality emphasizes the key scaling of the line-force with the velocity gradient dv/dr and *inverse* of the density, $1/\rho$. This keeps the line acceleration less than gravity in the dense, nearly static atmosphere, but also allows its outward increase above gravity to drive the outflowing wind.

The CAK mass loss rate is set by the associated critical density that allows the outward line acceleration to be just sufficient to overcome the (electron-scattering-reduced) gravity, i.e. with $\Gamma_{lines} \approx 1 - \Gamma_e$,

$$\dot{M}_{CAK} = \frac{\alpha}{1-\alpha} \frac{L}{c^2} \left[\frac{Q\Gamma_e}{1-\Gamma_e} \right]^{-1+1/\alpha} , \tag{16}$$

where we have used the definition of the mass loss rate $\dot{M} \equiv 4\pi\rho v r^2$ and the fact that for such a CAK solution, $vdv/dr \approx g(1-\Gamma_e)$.

This last property further yields the characteristic CAK velocity law scaling $v(r) \approx v_\infty (1-R/r)^{1/2}$, with the wind terminal speed being proportional to the effective surface escape speed,

$$v_\infty \propto v_{\rm eff} \equiv \sqrt{GM(1-\Gamma_e)/R} . \tag{17}$$

As a star approaches the classical Eddington limit $\Gamma_e \rightarrow 1$, these standard CAK scalings formally predict the mass loss rate to diverge as $\dot{M} \propto 1/(1-\Gamma_e)^{(1-\alpha)/\alpha}$, but with a vanishing terminal flow speed $v_\infty \propto \sqrt{1-\Gamma_e}$. The former might appear to provide an explanation for the large mass losses inferred in LBV's, but the latter fails to explain the moderately high inferred ejection speeds, e.g. the 500-800 km/s kinematic expansion inferred for the Homunculus nebula of η Carinae [30, 31].

So one essential point is that line-driving could never explain the extremely large mass loss rates needed to explain the Homunculus nebula. To maintain the moderately high terminal speeds, the $\Gamma_e/(1-\Gamma_e)$ factor would have to be of order unity. Then for optimal realistic values $\alpha = 1/2$ and $Q \approx 2000$ for the line opacity parameters [10], the maximum mass loss from line driving is given by [33],

$$\dot{M}_{max,lines} \approx 1.4 \times 10^{-4} L_6 \, M_\odot/yr \, , \tag{18}$$

where $L_6 \equiv L/10^6 L_\odot$. Even for peak luminosities of a few times $10^7 L_\odot$ during η Carinae's eruption, this limit is still several orders of magnitude below the mass loss needed to form the Homunculus. Thus, if mass loss during these eruptions occurs via a wind, it must be a super-Eddington wind driven by continuum radiation force (e.g., electron scattering opacity) and not lines [24, 5].

3.2 Continuum-driven winds regulated by porous opacity

As discussed in §2, stars that approach or exceed the Eddington limit are expected to have a complex spatial structure, and in recent years, this has lead to a new "porosity" paradigm [26, 27] for how continuum driving from super-Eddington stars can be regulated to yield a quasi-steady net mass loss. A key insight regards the fact that, in a laterally inhomogeneous atmosphere, the radiative transport should selectively avoid regions of enhanced density in favor of relatively low-density, "porous" channels between them. This stands in contrast to the usual picture of simple 1-D, gray-atmosphere models, wherein the requirements of radiative equilibrium ensure that the radiative flux must be maintained independent of the medium's optical thickness. In 2-D or 3-D porous media, even a gray, continuum opacity will lead to a flux avoidance of the most optically thick regions, much as in frequency-dependent radiative transfer in 1-D atmosphere, wherein the flux avoids spectral lines or bound-free edges that represent a localized spectral regions of non-gray enhancement in opacity.

A simple description of the effect is to consider a medium in which material has coagulated into discrete clumps of individual optical thickness $\tau_{cl} = \kappa \rho_b \ell$, where ℓ is the clump scale, and the clump density is enhanced compared to the mean density of the medium by a volume filling factor $f = \rho_b/\rho$. The effective overall opacity of this medium can then be approximated by a form very reminisent of the scaling of force from a single (cf. eq. 14),

$$\kappa_{\rm eff} \approx \kappa \, \frac{1 - e^{-\tau_{cl}}}{\tau_{cl}} \, . \tag{19}$$

Note that in the limit of optically thin clumps ($\tau_{cl} \ll 1$) this reproduces the usual microscopic opacity ($\kappa_{\rm eff} \approx \kappa$); but in the optically thick limit ($\tau_{cl} \gg 1$), the effective opacity is reduced by a factor of $1/\tau_{cl}$, thus yielding a medium with opacity characterized instead by the clump cross section divided by the clump mass ($\kappa_{\rm eff} = \kappa/\tau_{cl} = \ell^2/m_{cl}$). The critical mean density at which the clumps become optically thin is given by $\rho_o = 1/\kappa h$, where $h \equiv \ell/f$ is a characteristic "porosity length" parameter. A key upshot of this is that the radiative acceleration in such a gray, but spatially porous medium would likewise be reduced by a factor that depends on the mean density.

More realistically, it seems likely that structure should occur with a range of compression strengths and length scales. Noting the similarity of the single-scale and single-line correction factors (cf. eqns. 14 and 19), let us draw upon an analogy with the power-law distribution of line-opacity in the standard CAK model of line-driven winds, and thereby consider a *power-law-porosity* model in which the associated structure has a broad range of porosity length h. As detailed by [22], this leads to an effective Eddington parameter that scales as

$$\Gamma_{\rm eff} \approx \Gamma \left(\frac{\rho_o}{\rho} \right)^{\alpha_p} \;\; ; \;\; \rho > \rho_o \, , \tag{20}$$

where α_p is the porosity power index (analogous to the CAK line-distribution power index α), and $\rho_o \equiv 1/\kappa h_o$, with h_o now the porosity-length associated with the *strongest* (i.e. most optically thick) clump.

In rough analogy with the "mixing length" formalism of stellar convection, let us assume this porosity length h_o scales with gravitational scale height $H \equiv a^2/g$. Then the requirement that $\Gamma_{\rm eff} = 1$ at the wind sonic point yields a scaling for the mass loss rate scaling with luminosity. For the canonical case $\alpha_p = 1/2$, this takes the form [22],

$$\dot{M}_{por} \approx 4(\Gamma - 1)\,\frac{L}{ac}\,\frac{H}{h_o} \tag{21}$$

$$\approx 0.004(\Gamma - 1)\,\frac{M_\odot}{\mathrm{yr}}\,\frac{L_6}{a_{20}}\,\frac{H}{h}\,. \tag{22}$$

The second equality gives numerical evaluation in terms of characteristic values for the sound speed $a_{20} \equiv a/20$ km/s and luminosity $L_6 \equiv L/10^6 L_\odot$. Comparision with the CAK scalings (16) for a line-driven wind shows that the mass loss can be substantially higher from a super-Eddington star with porosity-regulated, continuum driving. Applying the extreme luminosity $L \approx 20 \times 10^6 L_\odot$ estimated for the 1840-60 outburst of η Carinae, which implies an Eddington parameter $\Gamma \approx 5$, the derived mass loss rate for a canonical porosity length of $h_o = H$ is $\dot{M}_{por} \approx 0.32 M_\odot/yr$, quite comparable to the inferred average $\sim 0.5 M_\odot/yr$ during this epoch.

Overall, it seems that, together with the ability to drive quite fast outflow speeds (of order the surface escape speed), the extended porosity formalism provides a promising basis for self-consistent dynamical modeling of even the most extreme mass loss outbursts of Luminous Blue Variables, namely those that, like the giant eruption of η Carinae, approach the photon tiring limit.

3.3 Photon tiring in a simple super-Eddington wind model

Before discussing simulations of porosity models with base mass flux above the tiring limit, let us first examine simpler, analytic models of such photon tiring for continuum-driven winds in which the Eddington factor is assumed to have an *explicit* spatial dependence $\Gamma(r)$. Specifically, let us assume this increase outward from $\Gamma(r) < 1$ in a static interior, crossing unity at some surface radius R, which represents the sonic point point of a supersonic mass outflow. The density ρ_s and sound speed a at this point set the mass loss rate $\dot{M} = 4\pi R^2 \rho_s a$, but otherwise gas pressure terms have negligible effect in the further supersonic acceleration of the outflow. The steady-state equation of motion thus reduces to

$$v\frac{dv}{dr} \approx -\frac{GM(1-\Gamma(r))}{r^2} \;\; ; \;\; r \geq R. \tag{23}$$

Note that, unlike in the porosity models above, this equation of motion has no explicit dependence on density, implying that the resulting velocity law would be entirely independent of the amount of mass accelerated. More realistically, a given radiative luminosity can only accelerate a limited mass loss rate before the energy expended in accelerating the outflow against gravity would necessarily come at the expense of a notable reduction in the radiative energy flux itself. To take accout of this “photon tiring”, we simply reduce the radiative luminosity according to the gained kinetic and potential energy of the flow,

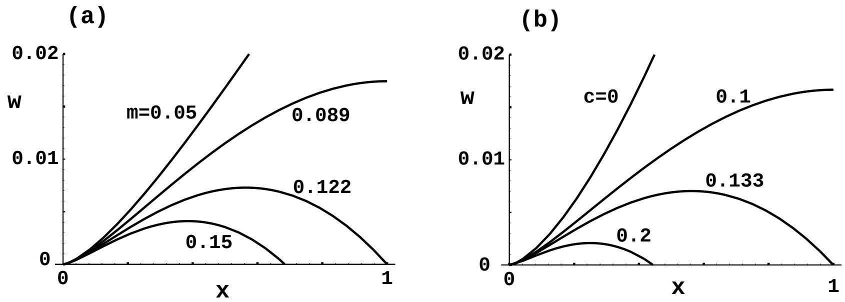

Fig. 3 Scaled kinetic energy w vs. scaled inverse radius $x = 1 - R/r$ in simple continuum-driven wind models illustrating flow stagnation from photon tiring (left), or a post-peak decline of opacity (right). The curves are labeled with the photon tiring number m, or opacity peak parameter c, defined in the text.

$$L = L_* - \dot{M}\left[\frac{v^2}{2} + \frac{GM}{R} - \frac{GM}{r}\right]. \tag{24}$$

Defining the scaled variables

$$w \equiv v^2 R/2GM\,; \quad x \equiv 1 - R/r, \tag{25}$$

we find the equation of motion with photon tiring can be written in the dimensionless form,

$$\frac{dw}{dx} = -1 + \Gamma(x)[1 - m(w + x)], \tag{26}$$

where the photon "tiring number",

$$m \equiv \frac{\dot{M}GM}{L_* R}\,, \tag{27}$$

characterizes the fraction of radiative energy lost in lifting the wind out of the stellar gravitational potential from R. Using integrating factors, it is possible to obtain an explicit solution to $w(x)$ in terms of the integral quantity $\bar{\Gamma}(x) \equiv \int_0^x dx'\Gamma(x')$,

$$w(x) = -x + \frac{1}{m}\left[1 - e^{-m\bar{\Gamma}(x)}\right] + w_o\,, \tag{28}$$

where for typical hot-star atmospheres the sonic point boundary value is very small, $w(0) = w_o \equiv a^2 R/2GM < 10^{-3}$.

As a simple example, consider the power law form $\Gamma(x) = 1 + 0.1\sqrt{x}$. Figure 3 plots solutions $w(x)$ vs. x for various m. For low m, the flow reaches a finite speed at large radii ($x = 1$), but for high m, it curves back, stopping at some finite *stagnation* point x_s, where $w(x_s) \equiv 0$. The latter solutions represent flows for which the mass loss rate is too high for the given stellar luminosity to be able to lift the material to full escape at large radii. By considering the critical case $w(x = 1) = 0$, we can define a maximum mass loss rate from m_{max}, given from eq. (28) by the transcendental relation,

$$m_{max} = 1 - e^{-m_{max}\bar{\Gamma}(1)} \approx 1 - e^{2-2\bar{\Gamma}(1)}, \tag{29}$$

where the last expression provides a good explicit approximation for any realistic $\bar{\Gamma}(1) > 1$.

Note that regardless of how large $\bar{\Gamma}(1)$ becomes, it is always true that $m_{max} < 1$, simply reflecting the fact that the mass loss is always limited by the rate at which the radiative luminosity can lift material out of the gravitational potential from R. By comparison, the maximum mass loss allowed by convective inefficiency [cf. eq. (10)] would correspond to a tiring number of order $m_{conv} \approx GM/Ra^2 \approx 2v_{esc}^2/a^2$. Thus since typically $m_{conv} \gg 1$, we conclude that any superEddington outflow initiated near the layer where convection becomes inefficient would generally stagnate by photon tiring well before any material could escape.

In the limit of negligible tiring $m \ll 1$, the flow solution (9) simplifies to

$$w(x) \approx \bar{\Gamma}(x) - x. \tag{30}$$

For a limited super-Eddington domain, the critical case of marginal escape with zero terminal velocity, $w(1) = 0$, is now set in general by the condition $\bar{\Gamma}(1) = 1$. Figure 3 illustrates solutions for the specific example of nonmonotonic $\Gamma(x) = 1 + 0.1\sqrt{x} - cx$, for various c. For all $\bar{\Gamma}(1) < 1$ (i.e., $c > 0.133$), the material stagnates at the radius where $\bar{\Gamma}(x_s) = x_s$, and so cannot escape the system in a steady state flow. In a time-dependent model, such material can be expected to accumulate at this stagnation radius, and possibly eventually fall back to the star. This represents another way in which, instead of a steady outflow, a limited superEddington region could give rise to an extended envelope with either a mass circulation or a density inversion.

3.4 Simulation of stagnation and fallback above the tiring limit

For porosity models in which the base mass flux *exceeds* the photon tiring limit, recent numerical simulations [39, 40] have explored the nature of the resulting complex pattern of infall and outflow. Despite the likely 3-D nature of such flow patterns, to keep the computation tractable, this initial exploration assumes 1-D spherical symmetry, though now allowing a fully time-dependent density and flow speed. The total rate of work done by the radiation on the outflow (or vice versa in regions of inflow) is again accounted for by a radial change of the radiative luminosity with radius,

$$\frac{dL}{dr} = -\dot{m} g_{rad} = -\kappa_{\rm eff}\, \rho v L/c\,, \tag{31}$$

where $\dot{m} \equiv 4\pi\rho v r^2$ is the local mass-flux at radius r, which is no longer a constant, or even monotonically positive, in such a time-dependent flow. The latter equality then follows from the definition (1) of the radiative acceleration g_{rad} for a gray

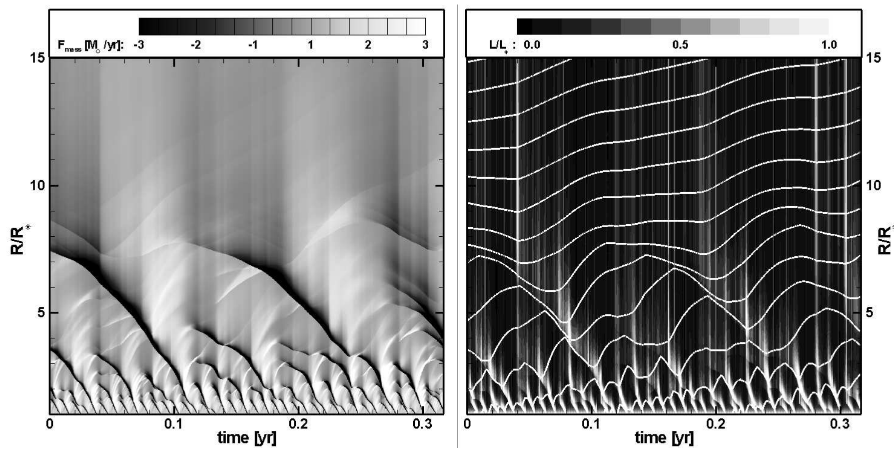

Fig. 4 Grayscale plot of radius and time variation of mass flux (left) and luminosity (right) in a time-dependent simulation of a super-Eddington wind with a porosity-mediated base mass flux above the photon tiring limit. The white contours on the right trace the height progression of fixed mass shells.

opacity $\kappa_{\rm eff}$, set here by porosity-modified electron scattering. At each time step, eq. (31) is integrated from an assumed lower boundary luminosity $L(R)$ to give the local radiative luminosity $L(r)$ at all radii $r > R$. Using this to compute the local radiative acceleration, the time-dependent equations for mass and momentum conservation are evolved forward to obtain the time and radial variation of density $\rho(r,t)$ and flow speed $v(r,t)$. (For simplicity, the temperature is fixed at the stellar effective temperature.) The base Eddington parameter is $\Gamma = 10$, and the analytic porosity mass flux is 2.3 times the tiring limit.

Figure 4 illustrates the flow structure as a function of radius (for $r =$ 1-15 R) and time (over an arbitrary interval long after the initial condition) The left panel grayscale shows the local mass flux, in $M_\odot/$ yr, with dark shades representing inflow, and light shades outflow. In the right panel, the shading represents the local luminosity in units of the base value, $L(r)/L(R)$, ranging from zero (black) to one (white); in addition, the superposed lines represent the radius and time variation of selected mass shells.

Both panels show the remarkably complex nature of the flow, with positive mass flux from the base overtaken by a hierarchy of infall from stagnated flow above. However, the re-energization of the radiative luminosity from this infall makes the region above have an outward impulse. The shell tracks thus show that, once material reaches a radius $r \approx 5R$, its infall intervals become ever shorter, allowing it eventually to drift outward. The overall result is a net, time-averaged mass loss through the outer boundary that is very close to the photon-tiring limit, with however a terminal flow speed $v_\infty \approx 50$ km/s that is substantially below the surface escape speed $v_{esc} \approx 600$ km/s.

These initial 1-D simulations thus provide an interesting glimpse into this competition below inflow and outflow. Of course, the structure in more realistic 2-D and 3-D models is likely to be even more complex, and may even lead itself to a highly porous medium. But overall, it seems that one robust property of super-Eddington stars may well be mass loss that is of the order of the photon tiring limit.

4 Discussion

4.1 LBV eruptions: enhanced winds or explosions?

A key theme of this book is that LBV eruptions can pose as "supernova imposters", since, like true supernovae, they are charcterized by a substantial brightening and a large ejection of mass. In this chapter, we have modeled this mass loss in terms of a quasi-steady, continuum-driven wind that results from the stellar luminosity exceeding the Eddington limit. But an alternative paradigm is that such eruptions might in fact be point-time "explosions" that simply did not have sufficient energy to completely disrupt the star.

Both paradigms require an unknown energy source, but one important distinction is that explosions are driven by *gas pressure*, whereas super-Eddington winds are driven by *radiation*. The two have markedly different timescales.

The overpressure from an explosion propagates through the star on a very short dynamical time scale, of order R/a, where a is the sound-speed in the very high temperature gas that is heated by the energy deposition of the explosion. In supernovae, this sound speed is on the order of the mass ejection speed, on the order of 10,000 km/s; even in a "failed" LBV explosion, it would be on the order of the surface escape speed, or a few hundred km/s, implying a dynamical time of order the free fall time, or just a few hours. Of course, the release of radiative energy is tied to the expansion (and later on, radioactive β-decay), and thus peaks on a somewhat longer time of a few days or weeks for supernovae. But it is difficult to see how such a direct gas-pressure-driven explosion could be maintained for the years to decade timescale inferred for LBV eruptions.

This then is perhaps the key argument for a radiation-driven model. If energy is released in the deep interior, its *radiative* signature can take up to a much longer *diffusion* time to reach the surface[3]. This can be long as a few years.

In contrast to the explosive disruption of supernovae, for LBV eruptions the total energy is typically well below the stellar binding energy. Thus even if this energy were released suddenly in the deep interior, the initial dynamical response would quickly stagnate, leaving then radiative diffusion as the fall-back transport. But since massive stars are already close to the Eddington limit, the associated excess lumi-

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[3] Since the luminous stars are likely to be mostly convective (e.g. §2.3), the limiting time scale is that of the convective diffusion's mixing length time in the stellar cores, which due to the high density is much longer than the dynamical time scales.

nosity should push it over this limit, leading then to the strong, *radiatively driven* mass loss described above.

Because this time scale is still much longer than any dynamical time in the system, the essential processes can be modeled in terms of a quasi-steady continuum-driven wind during this super-Eddington epoch, as described above.

4.2 Trigger & energy source for super-Eddington luminosity

Perhaps the least understood aspect of LBVs is the mechanism giving rise to the observed eruptions. In supernovae explosions, the energy source is obviously the core-collapse to a neutron star or black hole. But in LBV eruptions, the post-eruption survival of an intact star, and the indication at least some LBVs can undergo multiple giant eruptions, both show that the energy source cannot be a one-time singular event like core collapse. Some other mechanism must provide the energy.

The total energy associated with the eruption, which is composed of the radiative luminosity and mechanical (i.e., kinetic and potential) energy in the wind, is a few times 10^{49}erg. For an eruption lasting a few decades, this corresponds to a total luminsoity of a few times 10^{40}erg/sec, which is only a few times the Eddington luminosity. This suggests the energy for the eruption can be supplied by the nuclear burning itself. If during quiescence the star shines at the Eddington luminosity, all that is required is to increase the central temperature by about 10% for the CNO reactions to supply the extra energy. This has two implications.

First, the change in the binding energy required to get the increased temperature is only 10% of the binding energy of an $n = 3$ polytrope. For a ratio β for gas to total to radiation pressure, and and taking a mass $M \sim 100M_\odot$ and radius $R \sim 100R_\odot$, the energy change is $\sim 0.1 \times (1-\beta)GM^2/(2R) \sim 10^{49}$erg. Thus, in other words, the energy associated with the eruption is of order the change in the binding energy needed in order to supply the extra energy through CNO reactions. This implies that the whole star can participate in the eruption.

Second, because the energy for the eruption can be continuously generated, many different scenarios can be envisioned for the triggering mechanism. Moreover, a mechanism originating in the envelope, which changes its properties yet forces the whole star to readjust, like a "geyser" [7], is equally favorable as a mechanism originating in the core. Thus, quite a few mechanisms were suggested to explain LBV eruptions. Here are several examples.

Glatzel and Kiriakidis [12] found, while using standard opacity tables, mode-coupling instabilities in a linear analysis of radial modes in massive stars. These unstable "strange" modes arise in the region where radiation can diffuse very quickly, that is, in the outer layers of the stars. Guzik et al. [13] then showed how such an instability can propagate inwards through the turning on of convection. Moreover, since convection takes a finite time to adjust due to its "inertia", super-Eddinton luminosities can arise, and with it, ejection of mass.

Other instabilities originating in the stellar envelopes were found by Stothers and Chin [38], who hypothesized that η Car is repeatedly encountering ionization-induced dynamical instability, or by Maeder [19], who found that super-Eddington layers could arise somewhat deeper in.

As a contrast, other instabilities could originate at the core. Guzik [14], for example, suggested that nonradial gravity mode oscillations grow slowly to an amplitude sufficient to cause an episode of mixing of hydrogen-rich material downward into hotter denser layers, which would generate a burst of nuclear energy release.

Because luminous stars have large radiative pressure support, they tend to be more loosely bound and so exhibit quite a few instabilities. This explains the plethora of suggested mechanisms. Unfortunately, however, none seems notably more favorable than the others.

It is yet unclear why LBVs erupt, and what sets the eruption time scale, the total ejected mass, or what determines the eruption repetition rate. In particular, there is currently no model which predicts these numbers.

4.3 Evolution near the Eddington limit

It is fair to say that the evolution of extremely massive stars is not yet understood. In particular, the appreciation for the potentially key evolutionary role of continuum-driven mass loss is still quite recent, and the super-Eddington conditions driving this mass loss depends on instabilities that are still quite poorly understood.

Nevertheless, because the mass loss rates associated with super-Eddington states are so extremely large, it is possible to discuss the average state of these objects without relying on details. Such huge mass loss rates simply cannot be sustained over the evolutionary time scales. Thus, once a star reaches a near-Eddington state, it must evolve without crossing the Eddington limit for too long, since the large mass loss serves as a negative feedback to reduce the luminosity of the star.

To see this behavior more quantitatively, consider the following simplified model. When a star is near the Eddington limit, it is radiation pressure dominated and convective. If we approximate it as an $n = 3$ polytrope, we can use Eddington's quartic relation to relate the mass of the object and the gas to total pressure ratio β,

$$\frac{M}{M_\odot} = 18.3\frac{(1-\beta)^{1/2}}{\beta\mu}\,, \tag{32}$$

where μ is the molecular weight of the star.

As the star evolves, its molecular weight increases. However, if the mass loss rates are small (e.g., from line-driven winds only), the mass remains nearly constant, and so the increase in μ forces a reduction in β. Note that without the effect of

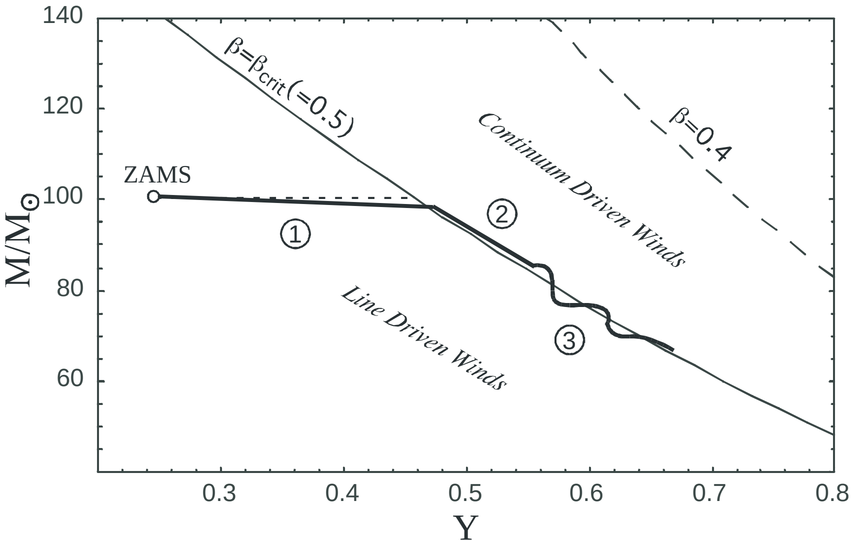

Fig. 5 The evolution of a very massive star in the Mass/Helium fraction plane. (1) As long as the star has a relatively high gas to total pressure ratio, it is below Eddington and will suffer only minor line driven mass-loss. (2) Once the molecular weight is high enough, it will reach the Eddington luminosity and start expelling continuum driven winds. Since these are generally very high, the evolution is forced to follow the Eddington luminosity from above, where moderate continuum-driven winds serve as a feedback keeping the Eddington luminosity. (3) In principle, the evolution along the Eddington luminosity can be uneventful, or, due to a yet unknown instability, include excursions to high luminosities and mass-loss rates, and quiescent periods in between (such that the *average* mass loss rate allows the star to follow $L_{\rm edd}$). The large eruption of η Car implies that its evolution includes excursions to high-mass loss rates. Note that we assumed here $\beta_{crit} = 0.5$. However, this number is actually unknown as it depends on the nonlinear state of the instabilities making atmospheres become porous.

porosity, the reduction in β forces the luminosity of the star to increase and approach the Eddington luminosity without actually reaching it.

However, for large enough radiation to gas pressure ratios, (i.e., small enough β), we know that instabilities should make the atmosphere porous and allow for a super-Eddington flux. That is, for a finite value of $\beta = \beta_{crit}$ the star will reach the Eddington luminosity, while for $\beta < \beta_{crit}$, it will be super-Eddington and result in a continuum-driven wind. According to eq. (22), this mass loss will evaporate the star on a time scale much shorter than its few million year lifetime, unless $\Gamma - 1 \sim \mathscr{O}(10^{-2})$, i.e., just above the Eddington luminosity.

Furthermore, as the evolution progresses, the increased molecular weight will force β to decrease. However, now that the star is just above Eddington, any decrease in β will drive a large mass loss. In other words, once the star reaches the Eddington limit, it will be forced to constantly lose mass in order to keep itself near the Eddington limit, i.e., following the quartic relation while keeping $\beta \approx \beta_{crit}$. This is portrayed in figure 5.

This evolution can continue as long as Hydrogen is burning in the core, and likely also after Hydrogen is depleted. This is because the remaining WR star should be exposed to similar constraints. The difference is that because μ is higher for Helium stars, the typical masses are smaller. This implies that towards the end phase of hydrogen burning, the remaining core will be typically much lighter than for a ZAMS star.

5 Summary

The very large mass loss rate observed in LBVs, and in η-Carinae in particular, strongly suggest that continuum driven winds are playing an important role. This is because continuum driving is the only mechanism known to give steady state winds with a mass loss rate much larger than line driving can under any conditions.

A continuum-driven wind, however, can only operate in a system above the Eddington luminosity. This implies that such a star should have a rather unique structure. Its "static" part should be composed of a large convective region, encompassing most of the star, and a *porous layer*, where inhomogeneities are responsible for a lowered opacity. At the radius where the opacity cannot be sufficiently reduced, there is a transition between the gravitationally bound layer below, and the radiatively accelerated wind above. Because the mass loss rate can be very large, the wind itself optically thick and the photosphere resides within the wind.

Another peculiar aspect of continuum driven winds is that because the mass loss rate is determined from conditions at the "static" surface of the star, the wind conditions are blind to whether the available radiative luminosity is sufficient to actually drive the wind to infinity. If the potential well is too deep, the result is that of *photon tired winds*, where the wind stagnates and falls back.

Current 1-D simulation of photon-tired winds suggest that the net results of these winds is to develop a layer with a hierarchical structure of shocks and sonically moving gas, allow for a large kinetic "luminosity", without driving mass above the photon-tired limit. It is not unreasonable that η Carinae actually reached this state, since the ejected mass divided by the 20 years of the giant eruption is, to within the rather large observational uncertainty, comparable to the photon-tired mass loss rate.

Although the super-Eddington states with continuum-driven winds can be stationary on a dynamical timescale, the large mass loss rates should substantially alter the stellar structure on an evolutionary timescale. As such, stellar structure and evolution should effectively control the overall level of mass loss, implying that mass loss and evolution are inextricably linked. We thus expect very massive stars that reach the Eddington limit (which they do once the hydrogen mass fraction decreases sufficiently) should evolve with a luminosity kept slightly above the Eddington value, with a relatively modest continuum driven wind.

Perhaps the key open questions still evading a solution regards the origin of the variability of LBVs and their giant mass loss episodes. While there is a general understanding of how a super-Eddington state with the large mass loss can exist, but there is yet no theory that predicts, for example, the two-decade-long giant eruption of η-Carina.

Acknowledgements S.P.O. acknowledges partial support of NSF grant AST-0507581 and N.J.S the support of ISF grant 1325/06. We thank J. MacDonald, N. Smith, R. Townsend, and A.J. van Marle for many helpful discussions.

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