Radiatively Driven Stellar Winds and Aspherical Mass Loss

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Abstract.
The high luminosity of massive stars can drive substantial mass loss, with significant consequences for their evolution and ultimate fate. A key general issue regards the relative importance of the cumulative loss in the comparitively moderate, *line*-driven stellar winds that persist through much of the star's evolution, versus briefer episodes of much stronger, *continuum*-driven mass loss associated with Luminous Blue Variable (LBV) phases, when the star may approach or exceed the Eddington limit. Building upon the standard CAK formalism for line-driven winds, the presentation here summarizes recent work on how the lateral structuring, or *porosity*, of a medium can moderate continuum driving, and lead to a much stronger mass loss that, as inferred for the giant eruption of eta Carina, approaches the "photon tiring" limit. A particular focus is the role of rapid (near-critical) stellar rotation in inducing an equatorial gravity darkening, with the associated polar brightening then driving both a higher polar mass flux and higher polar flow speed, a configuration that fits naturally with the inferred mass distribution and bipolar shape of the eta Carinae Homunculus nebula.

1. Instroduction

The evolution and fate of massive stars can be strongly influenced by the extensive mass loss that results from driving by their very large, sometimes super-Eddington luminosity. A particularly extreme example is eta Carinae, which during the giant eruption of 1840-60 is estimated to have ejected some $10M_\odot$ at moderately high speeds of 500-800 km/s, implying a wind kinetic energy loss rate that rivals the extreme radiative luminosity $L \approx 20 \times 10^6 L_\odot$ estimated for this epoch. Such extreme mass loss near the energy or "photon tiring" limit is orders of magnitude greater than can be readily explained by the standard CAK (Castor, Abbott, and Klein 1975) model for radiative driving by scattering in a ensemble of lines from metal ions. Instead, the observational association of eta Carinae and other "Luminous Blue Variables" (LBVs) as being near an apparent upper limit in luminosity for observed stars (Humphreys & Davidson 1979, 1984) has led to the general view that such strong episodes mass loss may stem from the star approaching or exceeding the Eddington limit, at which even the continuum force associated with just electron scattering exceeds the inward force of gravity.

The presentation here reviews the basic physics of radiative driving, focussing first on the standard CAK formalism for steady, line-driven stellar winds, and then using this as a basis to summarize recent models of super-Eddington mass loss driven by continuum scattering that is moderated by the "porosity" of a laterally structured medium. A particular emphasis is on the potential role of rapid rotation in inducing the aspherical, bipolar form inferred for the mass loss from eta Carinae.

2. Radiative Force and the Eddington Limit

As a basis for understanding radiative driving, let us begin by considering the general form for the radiative acceleration $\mathbf{g}_{rad}$ associated with a specific opacity κ_ν (a.k.a. the mass absorption coefficient, with CGS units cm^2/g) for interaction of stellar material with a radiative flux $\mathbf{F}_\nu$ at photon frequency ν,

$$\mathbf{g}_{rad} = \int_0^\infty d\nu \, \kappa_\nu \mathbf{F}_\nu / c \, , \tag{1}$$

with c the speed of light. In general the opacity κ_ν includes both broad-band continuum processes – e.g. Thomson scattering of electrons, and bound-free or free-free absorption – and bound-bound transitions associated with line absorption and/or scattering.

In the static envelope and atmosphere, the reduction in flux in saturated lines keeps the associated line-force small, and so to a good approximation the overall radiative acceleration is dominated by a continuum contribution characterized by electron scattering. Because such opacity is gray (frequency-independent), it can be pulled outside the frequency integration in eqn. (1). In an idealized, spherically symmetric, radiative envelope, the bolometric flux $F \equiv \int_0^\infty d\nu F_\nu$ is thus purely radial, and at any local radius r is simply set by $F = L/4\pi r^2$, where L is the total bolometric luminosity generated in the stellar core. The radiative acceleration associated with such a gray opacity κ is thus given by

$$g_{rad} = \kappa F/c = \frac{\kappa L}{4\pi r^2 c} \equiv \Gamma g \, , \tag{2}$$

where the last equality introduces the Eddington parameter $\Gamma \equiv \kappa L/4\pi GMc$, which gives the ratio of the radiative acceleration to the local gravitational acceleration, $g \equiv GM/r^2$, with G the gravitation constant and M the stellar mass.

A key point is that, since both gravity and radiative flux have the same r^{-2} decline with radius r, this ratio can often be considered nearly independent of radius, that is when the opacity κ, radiative luminosity L, and mass M are all fixed. (As discussed below, there are various circumstances in which this is not the case.) For the classical case of pure electron scattering, the Eddington parameter scales as

$$\Gamma_e = 2.6 \times 10^{-5} \frac{L}{L_\odot} \frac{M_\odot}{M} \tag{3}$$

Because stellar luminosity generally scales with a high power of the stellar mass, i.e. $L \propto M^{3-4}$, massive stars with $M > 10 M_\odot$ generally have electron Eddington parameters of order $\Gamma_e \approx 0.1 - 1$. Indeed, $\Gamma_e \equiv 1$ defines an *Eddington limit*, for which the entire star would become unbound, at least in this idealized model of 1-D, spherically symmetric, radiative envelope.

It should thus be emphasized, however, that this does not represent an appropriate condition for the steady-state mass loss characteristic of a stellar wind, since that requires an outwardly increasing radiative force that goes from being less than gravity in a bound stellar envelope to exceeding gravity in the outflowing stellar wind. In the next sections, we summarize how the necessary force modulation can occur through line-desaturation for line driving, and through porosity of spatial structure for continuum driving.

3. Line-Driven Stellar Winds

The resonant nature of line (bound-bound) absorption leads to an opacity that is inherently much stronger than from free electrons. For example, in the somewhat idealized, optically thin limit that all the line opacity could be illuminated with a flat, unattenuated continuum spectrum with the full stellar luminosity, the total line-force would exceed the free-electron force by a huge factor, of order $Q \approx 2000$ (Gayley 1995). For massive stars with typical electron Eddington parameters within a factor two of unity, $\Gamma_e \approx 1/2$, this implies a net outward line acceleration that could be as high as $\Gamma_{lines} \approx Q\Gamma_e \approx 1000$ times the acceleration of gravity!

Of course, this does not generally occur in practice because of the self-absorption of the lines. For a single line with integrated opacity $\kappa_q = q\kappa_e$, the reduction in the optically thin line-acceleration $q\Gamma_e$ can be written as

$$\Gamma_{line} \approx q\Gamma_e \, \frac{1 - e^{-qt}}{qt} \, , \tag{4}$$

where $t \equiv \kappa_e \rho c/(dv/dr)$ is the Sobolev optical depth of a line with unit strength, $q = 1$ (Sobolev 1960; CAK). Within the standard CAK line-driven wind theory, the number distribution N of spectral lines is approximated as a power law in line strength $q\, dN/dq = [1/\Gamma(\alpha)](q/Q)^{\alpha-1}$, where the CAK power index $\alpha \approx 0.5-0.7$ (and $\Gamma(\alpha)$ here represents the complete Gamma function). The associated line-ensemble-integrated radiation force is then reduced by a factor $1/(Qt)^\alpha$ from the optically thin value,

$$\Gamma_{lines} = \frac{Q\Gamma_e}{(1-\alpha)(Qt)^\alpha} \propto \left(\frac{1}{\rho} \frac{dv}{dr} \right)^\alpha \, . \tag{5}$$

The latter proportionality emphasizes the key scaling of the line-force with the velocity gradient dv/dr and *inverse* of the density, $1/\rho$. This keeps the line acceleration less than gravity in the dense, nearly static atmosphere, but also allows its outward increase above gravity to drive the outflowing wind. The CAK mass loss rate is set by the associated critical density that allows the outward line acceleration to be just sufficient to overcome the (electron-scattering-reduced) gravity, i.e. with $\Gamma_{lines} \approx 1 - \Gamma_e$,

$$\dot{M}_{CAK} = \frac{\alpha}{1-\alpha} \, \frac{L}{c^2} \left[\frac{Q\Gamma_e}{1-\Gamma_e} \right]^{-1+1/\alpha} \, , \tag{6}$$

where we have used the definition of the mass loss rate $\dot{M} \equiv 4\pi\rho v r^2$ and the fact that for such a CAK solution, $v dv/dr \approx g(1-\Gamma_e)$.

This last property further yields the characteristic CAK velocity law scaling $v(r) \approx v_\infty (1 - R/r)^{1/2}$, with the wind terminal speed being proportional to the effective surface escape speed,

$$v_\infty \propto v_{eff} \equiv \sqrt{GM(1-\Gamma_e)/R} \, . \tag{7}$$

As a star approaches the classical Eddington limit $\Gamma_e \rightarrow 1$, these standard CAK scalings formally predict the mass loss rate to diverge as $\dot{M} \propto 1/(1-\Gamma_e)^{(1-\alpha)/\alpha}$, but with a vanishing terminal flow speed $v_\infty \propto \sqrt{1-\Gamma_e}$. The former might appear to provide an explanation for the large mass losses inferred in LBV's, but the latter fails to explain the moderately high inferred ejection speeds, e.g. the 500-800 km/s kinematic expansion inferred for the Homunculus nebula of η Carinae (Smith 2002, Smith et al. 2003a).

Moreover, of course, such a divergence of the mass loss rate is precluded by the finite energy available in the stellar luminosity L, which sets a so-called "photon-tiring" limit for lifting mass out of the gravitational potential from the stellar surface (Owocki and Gayley 1997),

$$\dot{M}_{tir} = \frac{L}{v_{esc}^2/2} = \frac{L}{GM/R} = 0.032 \, \frac{M_\odot}{\mathrm{yr}} \; L_6 \; \frac{R/M}{R_\odot/M_\odot} \, , \tag{8}$$

where $L_6 \equiv L/10^6 L_\odot$. Comparison with eqn. (6) shows that photon tiring would limit CAK winds whenever

$$1 - \Gamma_e < \bar{Q}\Gamma_e \left[\frac{\alpha}{1-\alpha} \, \frac{v_{esc}^2}{2c^2} \right]^{\alpha/(1-\alpha)} \, . \tag{9}$$

4. Bipolar CAK Wind from Rotating, Gravity-Darkenend Star

The above results for a spherical wind can be readily adpated to derive approximate scalings for the latitudinal variation of mass flux and flow speed from a star undergoing a rapid enough rotation for the outward centrifugal acceleration to reduce significantly the effective surface gravity,

$$g_{eff}(\theta) = \frac{GM}{R^2} \, (1 - \Gamma_c - \Omega \sin\theta) \; . \tag{10}$$

Here θ is the stellar colatitude, and $\Omega \equiv v_{rot}^2 R/GM$, with v_{rot} the surface rotation speed. Using this to generalize the mass loss scaling in eqn. (6), we derive for the latitudinal dependence of the mass flux,

$$\dot{m}(\theta) \propto F(\theta)^{1/\alpha} g_{eff}(\theta)^{1-1/\alpha}, \tag{11}$$

where $F(\theta)$ gives the latitudinal dependence of the radiative flux. For example, for $\alpha = 1/2$, we find the scaling

$$\dot{m}(\theta) \;\; \propto \;\; \frac{1}{g_{eff}(\theta)} \;\; ; \;\; F(\theta) = const. \tag{12}$$

$$\propto \;\; g_{eff}(\theta) \;\; ; \;\; F(\theta) \propto g_{eff}(\theta). \tag{13}$$

The first form corresponds to the original result of Friend & Abbott (1986), who implicitly assumed the radiative flux is constant in latitude, and so concluded that the mass flux, and thus density, would be maximum near the stellar equator, where the effective gravity is most reduced by the centifugal force of the stellar rotation.

By contrast, the latter form assumes a standard von Zeipel (1924) *gravity-darkening law* in which the radiative flux itself scales in proportion to the effective surface gravity. As originally pointed out by Owocki, Cranmer, & Gayley (1996), this leads to a quite different, somewhat surprising result, namely that the mass flux should scale *directly* with effective gravity, and thus be at a *minimum* near the equator, and maximum near the relatively bright poles!

For the wind flow speed, the effect of rotation can be a bit more complicated, involving the integrated driving within 2D models of the outflow and radiative flux. Full simulation results show, however, the terminal flow speed retains a rough scaling the centrifugally reduced, effective escape speed at the source latitude

$$v_\infty(\theta) \propto v_{esc}(\theta) \propto \sqrt{g_{eff}(\theta)}\,. \tag{14}$$

Both the higher polar mass flux and higher polar flow speed fit the inferred conditions for eta Carinae's present day wind (Smith et al. 2003b), as well as for the giant eruption (Smith 2002). The potential relevance for explaining the bipolar form of the Homunculus was first suggested by Owocki and Gayley (1997), with further discussion by Maeder and Meynet (2001), Maeder and Desjacques (2001), and Dwarkadas and Owocki (2002). Actually, the large mass of the Homunculus makes it much more likely to have been driven by continuum than line opacity; but, as we show below (§§6-8), porosity-moderated continuum-driving from rapidly rotating, superEddington star seems capable of explaining both the high flow speed and nearly tiring-limited mass loss, while still retaining the key scalings needed to reproduce the bipolar form of the Homunculus.

5. Convective Instability of a Super-Eddington Stellar Interior

It should first be emphasized that locally exceeding the Eddington limit need *not* necessarily lead to initiation of a mass outflow. As first shown by Joss, Salpeter, and Ostriker (1972), in the stellar envelope allowing the Eddington parameter $\Gamma \rightarrow 1$ generally implies through the Schwarzschild criterion that material becomes *convectively unstable.* Since convection in such deep layers is highly efficient, the radiative luminosity is reduced, thereby lowering the associated radiative Eddington factor away from unity.

This suggests that a radiatively driven outflow should only be initiated *outside* the region where convection is *efficient.* An upper bound to the convective energy flux is set by

$$F_{conv} \approx v_{conv}\, l\, dU/dr \lesssim a\, H\, dP/dr \approx a^3 \rho, \tag{15}$$

where v_{conv}, l, and U are the convective velocity, mixing length, and internal energy density, and a, H, P, and ρ are the sound speed, pressure scale height, pressure, and mass density. Setting this maximum convective flux equal to the total stellar energy flux $L/4\pi r^2$ yields an estimate for the maximum mass loss rate that can be initiated by radiative driving,

$$\dot{M} \leq \frac{L}{a^2} \equiv \dot{M}_{max,conv} = \frac{v_{esc}^2}{2a^2}\dot{M}_{tir} \, , \tag{16}$$

where the last equality emphasizes that, for the usual case of a sound speed much smaller than the local escape speed, $a \ll v_{esc}$, such a mass loss would generally be well in excess of the photon-tiring limit set by the energy available to lift the material out of the star's gravitational potential (see eqn. 8). In other words, if a wind were to originate from where convection becomes inefficient, the mass loss would be so large that it would use all the available luminosity to accelerate out of the gravitational potential, implying that any such outflow would necessarily stagnate at some finite radius. One can imagine that the subsequent infall of material would likely form a complex spatial pattern, consisting of a mixture of both downdrafts and upflows, perhaps even resembling the 3D cells of thermally driven convection.

Indeed, dating back to early work by Spiegel (1976; 1977) there have speculations that an atmosphere supported by radiation pressure would likely exhibit Rayleigh-Taylor-type instabilities associated with support of a heavy fluid by a lighter one, leading to formation of "photon bubbles". Recent quantitative stability analyses by Spiegel and Tao (1999) and by Shaviv (2001) do lead to the conclusion that even a simple case of a pure "Thomson atmosphere" – i.e. supported by Thomson scattering of radiation by free electron – would be subject to intrinsic instabilities for development of lateral inhomogeneities. The analysis by Shaviv (2001) suggests in particular that these instabilities share many similar properties to the excitation of strange mode pulsations (e.g., Glatzel 1994). For example, they are favored when radiation pressure dominates over gas pressure, in this case operating in an intermediate regime between purely adiabatic and isothermal limits for the energy transport, wherein radiation diffuses against the opacity that shields localized gas compressions. With the dominance of radiation pressure, there is a tendency for it to compress the gas further, leading to an unstable growth of lateral structure. Shaviv (2001) identifies both stationary and propagating modes with maximum growth occurring at lateral scales comparable to the vertical scale height.

Overall, it seems that a star that exceeds the Eddington limit is likely to develop a complex spatial structure, whether due to local instability to convection, to global instability of flow stagnation, or to intrinsic compressive instabilities arising from the dominance of radiation pressure.

6. SuperEddington Outflow Moderated by Porous Opacity

Shaviv (1998; 2000) has applied these notions of a spatially structured, radiatively supported atmosphere to suggest an innovative paradigm for how quasi-stationary wind outflows could be maintained from objects that formally exceed the Eddington limit. A key insight regards the fact that, in a laterally inhomogeneous atmosphere, the radiative transport should selectively avoid regions of enhanced density in favor of relatively low-density, "porous" channels between them. This stands in contrast to the usual picture of simple 1D, gray-atmosphere models, wherein the requirements of radiative equilibrium ensure that the radiative flux must be maintained independent of the medium's optical thickness. In 2D or 3D porous media, even a gray opacity will lead to a flux avoidance of the most optically thick regions, much as in frequency-dependent radiative transfer in 1D atmosphere, wherein the flux avoids spectral lines or bound-free edges that represent a localized spectral regions of non-gray enhancement in opacity.

A simple description of the effect is to consider a medium in which material has coagulated into discrete blobs of individual optical thickness $\tau_b = \kappa \rho_b l$, where l is the blob scale, and the blob density is enhanced compared to the mean density of the medium by a volume filling factor $\rho_b/\rho = (\mathcal{L}/l)^3$, where $\mathcal{L}$ is the interblob spacing. The effective overall opacity of this medium can then be approximated as

$$\kappa_{eff} \approx \kappa \, \frac{1 - e^{-\tau_b}}{\tau_b} . \tag{17}$$

Note that in the limit of optically thin blobs ($\tau_b \ll 1$) this reproduces the usual microscopic opacity ($\kappa_{eff} \approx \kappa$); but in the optically thick limit ($\tau_b \gg 1$), the effective opacity is reduced by a factor of $1/\tau_b$, thus yielding a medium with opacity characterized instead by the blob cross section divided by the blob mass ($\kappa_{eff} = \kappa/\tau_b = l^2/m_b$). The critical mean density at which the blobs become optically thin is given by $\rho_o = 1/\kappa h$, where $h = \mathcal{L}^3/l^2$ is characteristic "porosity length" parameter.

A key upshot of this is that the radiative acceleration in such a gray, but spatially porous medium would likewise be reduced by a factor that depends on the mean density. In particular, for all $\rho(r) > \rho_o$, the effective Eddington factor would be given by $\Gamma_{eff} \approx \Gamma \rho_o/\rho$. Thus, for a formally super-Eddington atmosphere $\Gamma > 1$, an effective transition from stratified medium to quasi-steady wind outflow could now occur where $\Gamma_{eff} = 1$, representing a flow sonic-point with density $\rho_s = \Gamma \rho_o = \Gamma/\kappa h$. In terms of the sound speed a at this resulting sonic radius $r = R$, the associated porosity-moderated mass loss rate is given by

$$\dot{M}_{por} = 4\pi R^2 a \Gamma/\kappa h \tag{18}$$

$$= \frac{L}{ac} \frac{H}{h} \tag{19}$$

$$= 0.001 \, \frac{M_\odot}{\mathrm{yr}} \, \frac{L_6}{a_{20}} \, \frac{H}{h} . \tag{20}$$

In rough analogy with the "mixing length" formalism of stellar convection, the second equality relates the porosity length h to the gravitational scale height $H \equiv a^2/g$, yielding then a direct estimate for the mass loss rate scaling with luminosity L.

The third equality gives numerical evaluation in terms of characteristic values for the sound speed $a_{20} \equiv a/20$ km/s and luminosity $L_6 \equiv L/10^6 L_\odot$. Comparision with the CAK scalings (6) for a line-driven wind shows that the mass loss can be substantially higher from a super-Eddington star with porosity-moderated, continuum driving. Nonetheless, even applying the extreme luminosity $L \approx 20 \times 10^6 \, L_\odot$ estimated for the 1840-60 outburst of eta Carinae, the derived mass loss rate for a canonical porosity length of $h = H$ is still only $0.02 M_\odot/yr$, more than an order of magnitude below the inferred average $\sim 0.5 M_\odot/yr$ during this epoch.

7. Power-Law Porosity

The above simple model assumes the entire medium consists of clumps with a single porosity length h. More realistically, it seems likely that structure should occur with a range of compression strengths and length scales. Noting the similarity of the single-scale and single-line correction factors (cf. eqns. 17 and 4), let us draw upon an analogy with the power-law distribution of line-opacity in the standard CAK model of line-driven winds, and thereby consider a *power-law-porosity* model in which the associated structure has a broad range of porosity length h. As recently detailed by Owocki, Gayley, and Shaviv (2004; hereafter OGS), this leads to an effective Eddington parameter that scales as

$$\Gamma_{eff} \approx \Gamma \left(\frac{\rho_o}{\rho} \right)^{\alpha} \quad ; \quad \rho > \rho_o \, , \tag{21}$$

where α_p is the porosity power index (analogous to the CAK line-distribution power index α), and $\rho_o \equiv 1/\kappa h_o$, with h_o now the porosity-length associated with the *strongest* (i.e. most optically thick) clump. For power indices $\alpha_p < 1$, this weaker density dependence means that for a given Eddington parameter Γ the sonic point condition $\Gamma_{eff} = 1$ now occurs at a deeper, denser layer. This leads to mass loss rates that are enhanced over the single-scale model by a factor that increases with the Eddington parameter as Γ^{-1+1/α_p}. (For example, for $\alpha_p = 1/2$, the enhancement is by a factor $4(\Gamma - 1)$). For lower α_p ($\approx 0.5 - 0.6$) and/or moderately large Γ ($> 3 - 4$), such models have mass loss rates that now approach the photon tiring limit, as indeed is inferred for the giant outburst that lead to the Homunculus nebula. Determining the wind velocity law and terminal speed thus now requires integration of the photon-tiring-corrected equation of motion. (See eqn. 82 of OGS.)

The contour lines in figure 1 summarize results for how the mass loss rate and terminal speed depend on the two basic parameters of a power-law porosity model, namely power index α_p and Eddington parameter Γ. The gray area represents the range of observationally inferred values for the 1840-60 eruption of eta Carinae that gave rise to the Homunculus nebula. For $\alpha_p \approx 0.5$ and $\Gamma = 3 - 4$, there is generally good agreement; however OGS note that to match also the estmated radiative luminosity of $\sim 20 \times 10^6 L_\odot$ requires the star to have a mass to radius ratio of just a third the solar value, to limit the photon tiring loss of radiative luminosity in driving this strong mass loss from the star's surface gravitational potential.

Overall, we conclude that, together with the ability to drive quite fast outflow speeds (of order the surface escape speed), the extended porosity formalism summarized here provides a promising basis for self-consistent dynamical modeling of even the most extreme mass loss outbursts of Luminous Blue Variables, namely those that, like the giant eruption of η Carinae, approach the photon tiring limit.

8. Rotational Shaping of Bipolar LBV Nebula

A further key feature of these scalings for porosity-moderated mass loss is that, like the CAK model described in §§3-4, they retain the key properties to give

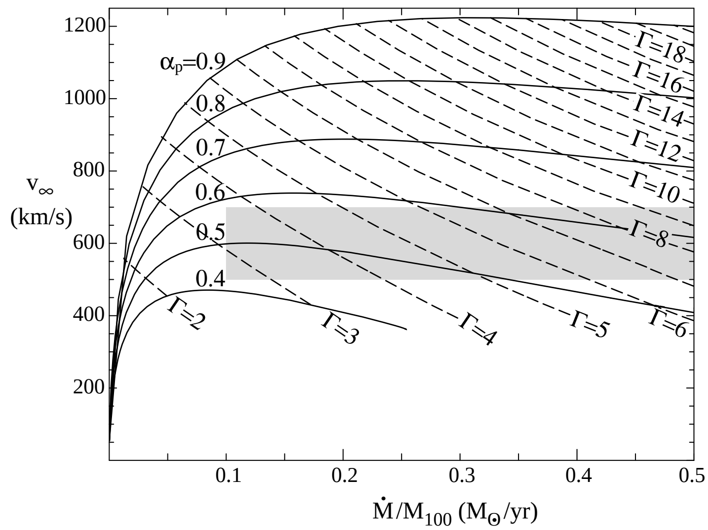

Figure 1. Results for power-law porosity model, plotted as contours for the power-index α_p (dashed) and Eddington parameter Γ (solid) in the plane defined by resulting mass loss rate ($\dot{M}$, scaled by stellar mass in units of $100\,M_\odot$) and terminal speed v_∞. The model also assumes a porosity length $h_o = H$ for the strongest clump, and a star with sound speed $a = 20\,\mathrm{km/s}$ and escape speed $v_{esc} = 620\,\mathrm{km/s}$ ($= v_{esc,\odot}$) at the stellar surface. The gray box denotes the observationally inferred parameter range for the 1840-60 giant eruption of η Carinae that gave rise to the Homunculus nebula.

a bipolar enhancement from a rapidly rotating, gravity-darkened source star. In particular, comparison of eqns. (6) and (18)-(20) shows that the mass loss rate in both cases scales with the stellar luminosity times a correction factor that is a function of the Eddington parameter, i.e., $\dot{M} \propto Lf[\Gamma]$. Accordingly, in a rotating star, the local surface mass flux $\dot{m}_{por}(\theta)$ at any colatitude θ should vary in proportion to the local surface radiation flux $F(\theta)$, times a function of the local effective Eddington parameter $\Gamma \equiv F/g_{\rm eff}$, where $g_{\rm eff}$ is the effective, centrifugally reduced surface gravity. But for the standard Von Zeipel (1924) gravity darkening scaling that $F(\theta) \propto g_{\rm eff}(\theta)$, we see that the Eddington parameter is *latitudinally constant*, implying then that the surface mass flux should again scale simply as (cf. eqn. 13)

$$\dot{m}_{por}(\theta) \propto F(\theta) \propto g_{\rm eff}(\theta)\,. \tag{22}$$

Since both the radiative flux and effective gravity are maximum at the rotational pole, eqn. (22) shows that the *mass flux* should be *strongest* near the *poles*.

Similar arguments can be made for the latitudinal variation of the flow expansion speed. Again, the detailed results may depend on latitudinal components of the mass flow, radiation flux, and radiative force, and should eventually be analyzed through 2-D models. But within the context of simple 1-D scaling relations for both line-driven and porosity models, the outflow speed should follow the same approximate scaling (14) as in the CAK model,

$$v_\infty(\theta) \propto v_{esc}(\theta) \propto \sqrt{g_{\rm eff}(\theta)}\,. \tag{23}$$

Since the effective gravity is highest toward the poles, we can expect the nebula expansion to be faster near the symmetry axis. Observations do indeed show that $v(\theta)$ for the polar lobes of the Homunculus nebula is roughly proportional to the simple latitudinal variation of escape speed from a rotating star (Smith 2002).

Overall, the expected faster polar flow speed can explain the generally prolate form of the expanding nebula, while the higher polar mass flux can explain the observationally inferred mass concentration near the polar symmetry axis. Thus, an attractive feature of the porosity-moderated continuum-driven formalism is that it preserves these key 1-D flow scalings from line-driven models, while allowing extension to much larger mass loss rates. However, as noted, more complete 2-D models should be developed to examine how these general scalings might be affected by latitudinal mass and radiation transport.

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