

Dynamics and Variability of Winds from Single WR Stars

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Abstract.

We review the dynamics of winds from single Wolf-Rayet (WR) stars, with emphasis on the following specific points: 1.) The classical “momentum problem” (to explain the large inferred ratio of wind to radiative momentum, $\eta \equiv \dot{M}v_\infty/(L/c) \gg 1$) is in principle readily solved through multiple scattering of radiation by an opacity that is sufficiently “gray” in its spectral distribution. In this case, one simply obtains $\eta \approx \tau$, where τ is the wind optical depth. 2.) Lines with a Poisson spectral distribution yield an “effectively gray” cumulative opacity, with multi-line scattering occurring when the velocity separation between thick lines Δv is less than the wind terminal speed v_∞ . In this case, one obtains $\eta \approx v_\infty/\Delta v$. 3.) However, realistic line lists are not gray, and leakage through gaps in the line spectral distribution tends to limit the effective scattering to $\eta \lesssim 1$. 4.) In WR winds, ionization stratification helps spread line-bunches and so fill in gaps, allowing for more effective global trapping of radiation, and thus $\eta > 1$. 5.) However, photon thermalization can reduce the local effectiveness of line-driving near the stellar core, making it difficult for radiation alone to initiate the wind. 6.) The relative complexity of WR wind initiation may be associated with the extensive turbulent structure inferred from observed variability in WR wind emission lines. 7.) Overall, the understanding of WR winds is perhaps best viewed as an “opacity problem”, i.e. identifying the enhanced opacity that can adequately block the radiation flux throughout the wind, and thus drive a WR mass loss that is much greater than from OB stars of comparable luminosity.

1. Introduction

One of the great, longstanding challenges in the study of Wolf-Rayet (WR) stars has been to understand the underlying causes and dynamics of their exceptionally strong stellar winds. Traditionally this challenge has been framed as a “momentum problem”, focussed on how a radiatively driven wind model could reproduce the very high wind “performance number”, $\eta \equiv \dot{M}v_\infty/(L/c)$, which represents the ratio of the radial momentum in the stellar wind, $\dot{M}v_\infty$, to that of the radiative luminosity, L/c . Whereas OB winds are within the

single-scattering limit $\eta \lesssim 1$ for a radiatively driven flow, inferred properties for WR winds generally imply performance numbers well in excess of this limit, typically exhibiting $\eta \approx 10$, with extreme cases ranging to $\eta \approx 50$ or even higher (Schmutz *et al.* 1989; Willis 1991; Hamann *et al.* 1995). Over the years, there have been many papers (Friend and Castor 1983; Abbott and Lucy 1986; Kato and Iben 1992; Lucy and Abbott 1993; Springmann 1994, 1998; Gayley, Owocki, and Cranmer 1995) examining how performance numbers above unity can in principle be obtained through multiple scattering of radiation.

Here we review the general physical requirements for achieving effective multiple scattering from a dense overlap of optically thick spectral lines, with special emphasis on the need to block the stellar flux by limiting leakage of radiation through gaps in the spectral distribution of strong wind lines. A general theme is to highlight the fundamental “opacity problem” of identifying the underlying source of the greater level of line-opacity needed to drive the much stronger mass loss of WR vs. OB stars.

2. Multiple Momentum Deposition

2.1. Example of a Static Gray Envelope

The total radial momentum imparted by radiation on a spherically symmetric circumstellar envelope can be expressed in terms of the radiative force density ρg_{rad} integrated over volume, outward from the wind base at radius $r = R$,

$$\dot{p}_{rad} = \int_R^\infty 4\pi r^2 \rho g_{rad} dr, \quad (1)$$

where the radiative acceleration is given by a frequency integral of the opacity κ_ν over the stellar flux spectrum, F_ν ,

$$g_{rad} = \int_0^\infty \frac{\kappa_\nu F_\nu}{c} d\nu \quad (2)$$

A particularly simple way to illustrate the requirements of multiple momentum deposition is in terms of an envelope with a *gray* opacity κ (Owocki and Gayley 1995; Gayley *et al.* 1994). In this case, the flux is just a constant in frequency, set at each radius by the bolometric luminosity through $F = L/4\pi r^2$. This implies $g_{rad} = \kappa L/4\pi r^2 c$, and so yields

$$\dot{p}_{rad} = \frac{L}{c} \int_R^\infty \kappa \rho dr = \frac{L}{c} \tau, \quad (3)$$

where τ is the total wind optical depth. We thus see that the requirement for exceeding the single scattering limit for radiative momentum deposition, $\dot{p}_{rad} > L/c$, is simply that the envelope be optically thick, $\tau > 1$.

Figure 1 provides a geometric illustration of how multiple momentum deposition occurs in an optically thick envelope. Figure 1a shows the case of a hollow shell with optical thickness $\tau = 5$, wherein a photon is backscattered within the hollow sphere roughly τ times before escaping, having thus cumulatively imparted τ times the single photon momentum, as given by eq. (3).

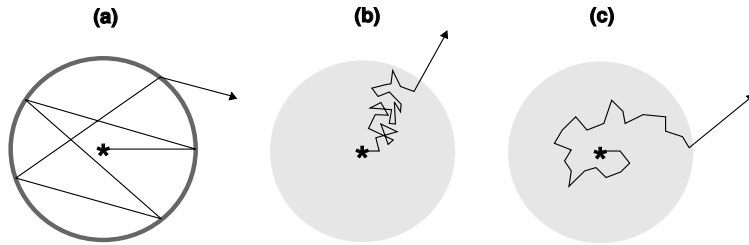


Figure 1. Multiscattered photon trajectories in (a) hollow and (b) filled gray spheres with the same central optical $\tau = 5$. The latter is shown ‘untangled’ in part (c) to illustrate that the multiple momentum deposition associated with hemispheric crossing in (a) is analogous to an effective ‘winding’ of the photon diffusion in (b).

However, the same momentum deposition also occurs in a *solid* sphere with the same total optical depth, even though, as shown in figure 1b, photons in this case undergo a much more localized diffusion without hemispheric crossing. Figure 1c illustrates how these diffusive vs. direct-flight pictures of multiple momentum deposition can be reconciled by thinking in terms of an effective “winding” around the envelope. For each scattering within a spherical envelope the radial momentum deposition is unchanged by arbitrary rotations about a radius through the scattering point, including rotations that bring the scattered trajectory into a single plane, with the azimuthal component always of the same sense, say clockwise viewed from above the plane. In this artificial construction, scattering thus leads to a *systematic* (vs. random walk) drift of the photon along one azimuthal direction, implying a cumulative *winding* trajectory for which the systematic outward push of the scattered photon is now apparent.

2.2. Multi-Line Transfer in an Expanding Wind

The multiple scattering by a dense ensemble of lines can actually be related to the above gray envelope case, *if* the spectral distribution of lines is Poisson, and so is spread throughout the spectrum without extended bunches or gaps. As first noted by Friend and Castor (1983), and later expanded on by Gayley *et al.* (1994), a wind driven by such an effectively gray line-distribution can be analyzed through an extension of the standard Castor, Abbott, and Klein (1975, hereafter CAK) formalism traditionally applied to the more moderate winds of OB stars. In the case of dense overlap, wherein optically thick lines in the wind have a frequency separation characterized by a velocity Δv much less than the wind terminal speed v_∞ , the mean-free-path between photon interactions with separate lines is given by

$$\frac{1}{\rho \kappa^{eff}} = \frac{\Delta v}{dv_i/dl}, \quad (4)$$

where dv_i/dl is the projected wind velocity gradient along a given direction of the photon pathlength l . The directional dependence of this velocity gradient implies an inherent *anisotropy* to the associated effectively gray line-ensemble opacity κ^{eff} . However, a full non-isotropic diffusion analysis (Gayley *et al.* 1994)

indicates that the overall wind dynamics is not too sensitive to this anisotropy. In particular, the total wind momentum can still be characterized by the effective radial optical depth, τ_r^{eff} , which in this case yields,

$$\dot{p}_{rad} \approx \frac{L}{c} \tau_r^{eff} = \frac{L}{c} \int_R^\infty \rho \kappa_r^{eff} dr = \frac{L}{c} \int_R^\infty \frac{1}{\Delta v} \frac{dv_r}{dr} dr = \frac{L}{c} \frac{v_\infty}{\Delta v}, \quad (5)$$

from which we identify $\tau_r^{eff} = v_\infty/\Delta v$. Neglecting a modest correction for gravitational escape, global momentum balance requires $\dot{p}_{rad} \approx \dot{M}v_\infty$, thus implying

$$\eta \approx \frac{v_\infty}{\Delta v}. \quad (6)$$

For winds driven by a gray ensemble of lines, we thus see that large performance numbers simply require that there be a large number of optically thick lines overlapping within the wind.

Note that eq. (6) implies that the mass loss scales as

$$\dot{M} \approx \frac{L}{c^2} \frac{c}{\Delta v}, \quad (7)$$

wherein we note that $c/\Delta v$ just represents the total, spectrum-integrated number of thick lines. It is important to realize, however, that this number of thick lines is not fixed *a priori*, but is itself dependent on wind properties like the mass loss rate and velocity law. Within the CAK formalism in which the line number distribution in opacity is given by a power law, one again finds that the mass loss follows a standard CAK scaling relation,

$$\dot{M} \approx \frac{L}{c^2} \left(\frac{Q\Gamma}{1-\Gamma} \right)^{(1-\alpha)/\alpha}, \quad (8)$$

where $\Gamma (\equiv \kappa_e L/4\pi GMc)$ is the Eddington parameter, α is the usual CAK exponent, and the line-opacity normalization constant Q is related to the CAK constant k by $k \sim Q^{1-\alpha}$ (Gayley 1995). Note that the second factor in eq. (8) simply represents the total number of thick lines, $c/\Delta v$.

As emphasized by Kudritzki *et al.* (1995), the CAK mass-loss law together with the tendency for the terminal speed v_∞ to scale with the effective escape speed $v_{esc} = \sqrt{2GM(1-\Gamma)/R}$ implies a wind-momentum-luminosity relation,

$$\dot{M}v_\infty\sqrt{R} \sim L^{1/\alpha}Q^{1/\alpha-1} \quad (9)$$

wherein we have neglected a residual dependence on $M(1-\Gamma)$ that is generally very weak for the usual case that α is near 2/3. Fits for galactic OB supergiants (Puls *et al.* 1996) give a luminosity slope consistent with $\alpha \approx 0.57$, with a normalization implying $Q \approx 10^3$.

For WR stars, such comparisons of wind momentum vs. luminosity show a much greater scatter, but with momenta consistently above those inferred for OB stars, typically reflecting more than a factor 10 higher mass loss rate for the same luminosity (Hamman *et al.* 1995). If we assume the same $\alpha \approx 0.6$ that characterizes OB winds, eq. (9) suggests that WR winds must have a line

opacity normalization Q that is more than a factor $\approx 10^{\alpha/(1-\alpha)} \approx 30$ higher! But this enhanced mass loss could also be obtained through a slightly lower α , representing a somewhat flatter number distribution in line opacity. For example, for stars with Eddington parameter $\Gamma \approx 1/2$ and O-star value for $Q \approx 10^3$, simply decreasing from $\alpha = 0.6$ to $\alpha = 0.5$ yields the required factor $Q^{1/0.5-1/0.6} \approx 10$ increase in \dot{M} .

While such a modest reduction in α may seem more plausible than a large increase in Q , it is generally not clear what basic properties of WR winds could lead to either type of change in the line opacity distribution. Viewed in this way, the fundamental distinguishing characteristic of WR winds is not so much their large performance number, but rather their enhanced mass loss compared to OB stars with similar luminosity. Identifying the sources of the enhanced opacity required to drive this enhanced mass loss thus represents a fundamental, unsolved “opacity problem”.

3. Momentum Deposition for a Non-Gray Line Distribution

3.1. Monte-Carlo and NLTE Models with Detailed Line Lists

Another essential complication for developing realistic models of WR winds stems from the inherently non-Poisson character of the spectral line distributions derived from atomic databases of line lists (Abbott and Lucy 1985; Lucy and Abbott 1993). As recently elaborated by Springmann (1998), at any given wind radius, the dominant contribution to the line opacity stems from a surprisingly small number of specific ionization stages of abundant heavy metals, primarily iron and iron-group elements. Moreover, for any given ion stage, the term structure is such that the lines are notably “bunched” into relatively restricted ranges of wavelength. With just a small number of distinct ion stages, the cumulative line spectrum thus likewise exhibits extensive wavelength bunching. Within the gaps between these bunches, the radiation can propagate relatively unimpeded by line-scattering, thus representing a potential preferential “leakage” that can significantly reduce the global radiative momentum deposition.

A good illustration of these bunches and gaps in the line spectral distribution is given by figure 2, taken from Springmann and Puls (1997). This shows contours of the number of thick wind lines as a function of wavelength and wind velocity, for both the canonical OB star Zeta Pup, and a model WN5 star. Because the ionization of an OB wind is nearly constant, the line bunches and gaps in the Zeta Pup case remain nearly fixed throughout the entire wind velocity range. By contrast, for the WR wind there is significant *ionization stratification*, and this leads to a mixing of the spectral positions of local bunches and gaps.

This characteristic ionization stratification of WR winds has been long known empirically through the observed inverse correlation between emission-line width and ionization potential (Beals 1941; Schulte-Ladbeck *et al.* 1995). But its potential significance for wind dynamics was first emphasized by Lucy and Abbott (1993), who pointed out that the associated tendency to fill in line-distribution gaps can substantially enhance the effective global blocking of radiative flux by multi-line scattering. Using detailed line-lists compiled from an atomic database, Lucy and Abbott carried out Monte-Carlo calculations of the transport of stellar core radiation through a WR wind with a fixed “beta=1”

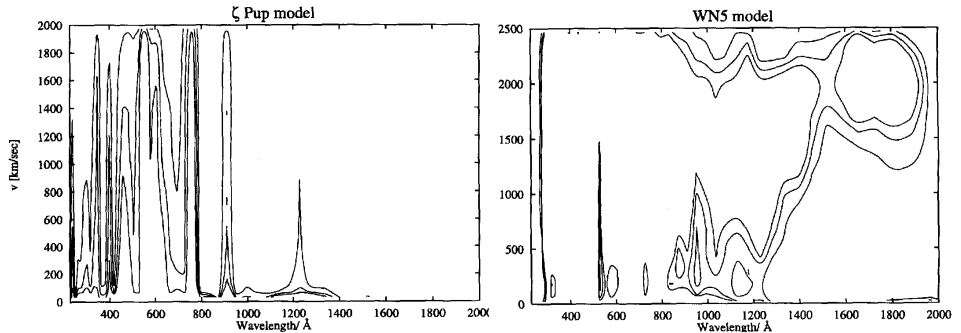


Figure 2. Contours of the number of thick wind lines as a function of wavelength and wind velocity, for both the canonical OB star Zeta Pup, and a model WN5 star. From Springmann and Puls (1997).

velocity law (i.e., $v(r) \sim (1 - R/r)^\beta$, with $\beta = 1$). After iterating to obtain the globally supported mass loss rate, they found that, for the specific WN5 star model considered, the radiative momentum deposition was sufficient to balance the *global* momentum requirements at a performance number of $\eta \approx 10$. However, they did not attempt to iterate to a full, *local* dynamical consistency, ending with a radiative acceleration that was too weak in the inner wind, and too strong in the outer wind, in both cases by up to about a factor two.

In these Monte Carlo models, a crucial simplification was to impose the ionization stratification semi-empirically by invoking an ionization radiation temperature whose inverse was taken to vary linearly with wind speed. From more fundamental non-LTE calculations, Schmutz (1997) has pointed out the crucial role of HeIII recombination in controlling, through the HeII Lyman continuum flux, the overall ionization. To obtain the He recombination needed to induce ionization stratification, he finds it necessary to introduce a “photon loss” in the HeII Ly- α transition. In this case, he finds models with WR mass loss rates do have sufficient radiative force to support the acceleration of the outer wind, but still lack sufficient driving in the inner regions (i.e., for $v < v_{esc}$).

Recent Ph.D. thesis research by U. Springmann has applied the Monte Carlo approach to investigate multi-line scattering both for the idealized, Poisson-distributed, effectively gray line-distribution case (Springmann 1994), as well as for more realistic models based on a detailed line list (Springmann 1998). The former represents an approach that nicely complements, in both a computational and conceptual sense, the semi-analytic, non-isotropic diffusion analysis by Gayley *et al.* (1994), with good overall agreement. The two methods, for example, independently led to the conclusion that multi-line scattering should cause a somewhat more extended outer wind acceleration, a result that at least trends in the right direction to explain the quite large “beta” velocity exponents inferred by observations (e.g., Robert 1991; Lepine 1998).

Springmann’s more recent calculations using a detailed line list are likewise in good general agreement with the previous results of Lucy and Abbott (1993), but are applied to an extensive grid of WR model parameters. The results show increased performance numbers for stars toward the upper left of the HR diagram, with a maximum of $\eta = 10.1$ for a star with $T_{eff} = 90kK$

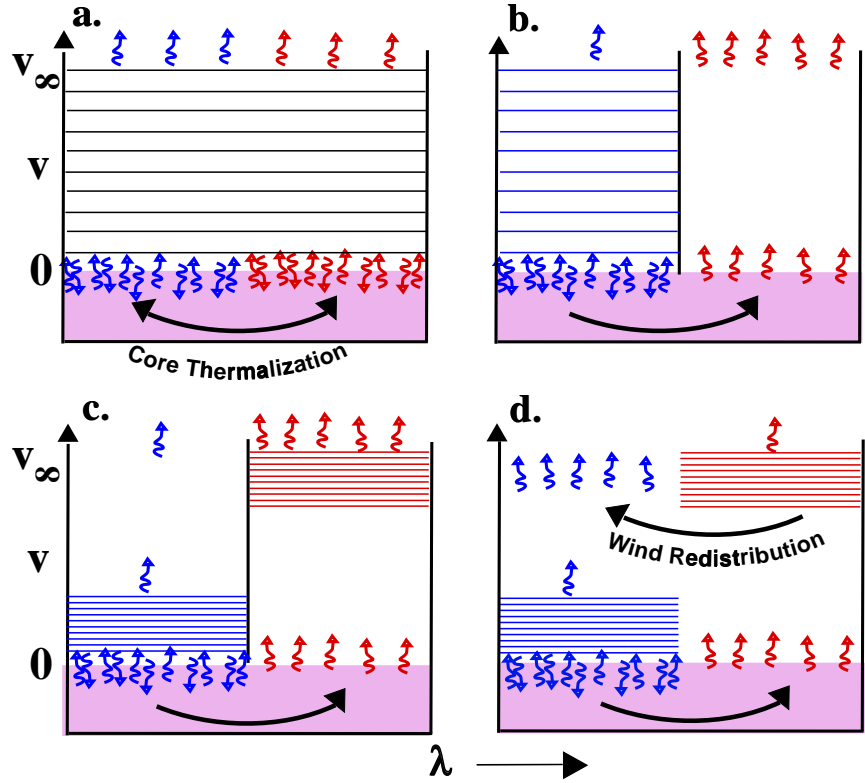


Figure 3. Schematic diagram illustrating the role of line gaps, line bunches, and photon thermalization for wind momentum deposition. As detailed in the text, the four parts represent: (a.) the effectively gray model; (b.) an OB wind with fixed ionization and extensive gaps; (c.) a WR wind with ionization stratification that fills gaps; (d.) the importance of limiting spectral redistribution in the wind.

and $\log(L/L_{\odot}) = 5.6$. This represents a distinct enhancement (by a factor ≈ 7) over the performance numbers corresponding to the empirical wind-momentum-luminosity relation derived for OB supergiants. However, comparison with performance numbers from observationally inferred data tabulated by Hamann *et al.* (1995) shows that the theoretical models are still systematically lower, typically by a factor 10. Some, but probably not all, of this remaining discrepancy may reflect overestimation of mass loss rates (e.g., by not properly accounting for wind clumping; Hillier 1991), or underestimation of the wind luminosity (e.g., by not properly accounting for line-blocking; Hillier and Miller 1998).

Overall it seems optically thick WR wind models with ionization stratification provide some, but not all, of the solution to both the WR momentum and opacity problems. Moreover, even within the enhanced global wind momentum supported in such models, there is locally insufficient radiative driving in the lower regions where the wind is initiated.

3.2. Role of Line Bunches, Gaps, and Core Thermalization

To provide a physical perspective to the above Monte Carlo and NLTE models, it is helpful to return to the concept from §2.2 that the line-ensemble constitutes an effective *continuum* opacity, but now allowing this to be frequency dependent to account for the relative bunches and gaps in the spectral distribution of lines. As seen from eq. (2), the radiative acceleration for such a non-gray opacity depends on the spectral integral of the opacity times the *frequency-dependent* radiation flux. In general the flux spectrum at any given location in an atmosphere or wind depends on self-consistent solution of a global radiation transport problem, with overall characteristics depending critically on the *thermalization* and frequency *redistribution* properties of the medium.

Figures 3 a-d illustrate this key role of photon thermalization and redistribution for simple non-gray line-distributions that are divided locally into two distinct spectral regions, representing either a line “bunch” or “gap”. Figure 3a first recaps the effectively gray case in which the entire spectrum is covered by lines at fixed velocity separation $\Delta v = v_\infty/10$. Since this represents a total effective optical depth $\tau = 10$ for photons to escape from the surface ($v = 0$) to infinity ($v = v_\infty$), the global radiative momentum is $\dot{p}_{rad} = 10L/c$, as follows from eq. (3).

Figure 3b represents the case when lines have the same concentration $\Delta v = v_\infty/10$ in half the spectrum (the bunch), with the other half completely line-free (the gap). Photons blocked in the bunched region are then rethermalized in the stellar core, and so tend to escape though the gap. Simple statistical arguments show that only about $0.5/(1 + \tau/2) = 1/12$ of the total flux now makes it out through the half of the spectrum covered by the bunch, implying from eq. (3) that the total radiative momentum is now only $\dot{p}_{rad}/(L/c) \approx 10 \times 1/12 + 0 \times 11/12 = 5/6 < 1!$ This roughly represents the circumstance applicable to OB star winds, which have radially constant ionization, and so a radially fixed line-spectrum. For such winds, we can see that any significant spectral gaps keep the performance number to near the single scattering limit $\dot{p}_{rad} \approx L/c$, even if there is very extensive line overlap in spectral line bunches.

Figure 3c represents the case wherein there are again $\tau = 10$ lines covering the full spectrum, but now over a limited spatial range, occurring very close to the star in one spectral region, and very far away in the other. In the region where line-blocking occurs near the star, core thermalization again channels photons to the other spectral region. But in this second spectral region, the blocking occurs far away from the star, with greatly reduced probability of backscattering to thermal redistribution in the stellar core. Again assuming purely coherent line-scattering in the wind, this means the flux is nearly independent of the outer wind blocking. Since all the stellar flux must diffuse through a layer somewhere with $\tau = 10$ lines, the global momentum deposition is now again, as in case a, simply $\dot{p}_{rad} = 10L/c$. However, note that the flux distribution is still similar to case b, i.e. in proportion 1/12 and 11/12 to the spectral regions corresponding respectively to the inner and outer wind blocking. This thus implies the same ratio for the relative deposition of radiative momentum. Overall we see that this example, intended to represent the case of an optically thick WR wind with ionization stratification, does indeed illustrate how it is possible to get a large global momentum deposition even when the line-opacity-spectrum is

locally divided into gaps and bunches. However, it also helps explain why, as found in the detailed models described in §3.1, this radiative momentum tends to be deposited more in the outer wind, leaving a net deficit in driving needed to initiate the outflow in the lower wind.

The final example in figure 3d illustrates the crucial importance of the assumption that the radiative transfer within the wind itself is through pure, coherent scattering. If there is significant thermalization or any other type of spectral energy redistribution within the wind itself, then radiation will always tend to be channeled into local gaps, thus again limiting the momentum deposition to a level roughly characterized by the single scattering limit.

3.3. Wind Initiation and Variability

Detailed monitoring of WR wind emission lines provides clear evidence for an extensive, stochastic, small-scale variability (Robert 1991; Lepine 1998). Such variability might develop from the known *intrinsic* instability of line-driving (Owocki 1994), but the limited analysis done so far (for the effectively gray case; see Gayley et al. 1996) indicates that the growth rate of this instability should be significantly reduced, by roughly a factor of the performance number, relative to the growth in OB winds. An alternative is thus that the wind variability is *induced* by the underlying star, e.g. by pulsations. Given the particular difficulties noted above in getting a sufficiently strong radiative force in the inner wind, it is even possible to consider a “two-stage” driving process, by which strong stellar pulsations actually initiate the mass loss (Glatzel *et al.* 1993), with radiative forces taking over to drive the extended acceleration and high terminal speeds. Indeed, for the slow, massive winds of cool AGB stars, a quite analogous two-stage process is the standard model, with pulsations lifting and extending material to a height where dust formation allows continued flow driving by radiation (Willson 1987; Netzer and Elitzur 1993).

Finally, even if radiative driving is by itself sufficient to initiate a WR wind, it seems plausible that the base outflow could be significantly more variable than in the O-star case. For O-stars the near radial-constancy of the ionization, and hence of the line-driving parameters, means there is a natural “compatibility” between the mass flux initiated in the inner wind, and the line-force available to drive material far from the star. In WR winds, the ionization stratification and the complexity of the photon blocking process (dependence on gaps, thermalization, etc.) means that such inner/outer wind compatibility is no longer guaranteed. In principle, this complexity might well allow for new kinds of global instabilities that would result in variable mass loss over a broad range of scales.

4. Conclusion

Much of the historical emphasis on a “momentum problem” for WR winds seems rooted in the intuitive notion that the momentum of a radiatively driven wind should be limited by the momentum of the radiation. In a fundamental sense, this notion is false, since we see that multiple scattering can, in principle, readily lead to multiple momentum deposition. In practice, however, the level of such multiple scattering is indeed limited by leakage through gaps in the spectral distribution of opacity, especially in the more thermalized lower regions where

the wind is initiated. In this context, it remains relevant to cite the large performance numbers of WR winds. But it seems the key questions are why WR stars have such a larger mass loss than OB stars of comparable luminosity, and what could cause the enhanced opacity needed to drive this enhanced mass loss.

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