

The Physics of Stellar Winds Near the Eddington Limit

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Abstract. We review the physics of sustained mass loss for stars near or exceeding the Eddington limit. We first emphasize that a superEddington condition need not generally lead to sustained mass loss, but instead can induce convection, a pressure and density inversion, or a stagnated outflow, all of which should have a net general effect of extending the stellar envelope. Next, we derive simple analytic flow solutions to illustrate the role of “photon tiring” or unsustainable driving in the deceleration and stagnation of an initial outflow. We then summarize how a continuum force near the Eddington limit can substantially enhance the mass loss in a line-driven outflow. Finally, we discuss the latitudinal variation of such line-driven mass loss in stars rotating near the critical, ‘Omega’ limit, noting in particular that the expected equatorial gravity darkening of the stellar radiation field can lead naturally to a prolate, bipolar wind outflow similar to that observed for η Car and other LBVs.

1. Introduction

What happens to a stellar wind as a star approaches the Eddington limit? The *classical* Eddington limit refers to the circumstance when the outward radiative force from scattering by free electrons approaches the inward force of gravity. Since both gravity and the free electron force have nearly the same radial dependence (i.e. as $1/r^2$), the ratio, defining the classical Eddington factor $\Gamma_e \equiv \kappa_e L / 4\pi GMc \approx 2.6 \times 10^{-5} (L/L_\odot) / (M/M_\odot)$, is nearly constant, fixed throughout the star by the luminosity/mass ratio L/M . Stellar wind models thus traditionally incorporate the effect of the electron scattering force simply by defining a reduced effective gravity or mass, $g_{eff} \sim M_{eff} \equiv M(1 - \Gamma_e)$. The scalings derived from standard CAK (Castor, Abbott, & Klein 1975) line-driven wind theory thus imply that approaching the classical Eddington limit $\Gamma_e \rightarrow 1$ should lead to a markedly enhanced mass loss rate, as well as a reduced wind terminal speed (see §4 below).

Surprisingly though, for LBVs the classical Eddington factors are typically no larger than for “normal” OB supergiants, i.e. $\Gamma_e \sim 0.5$. However, their location in the H-R diagram suggests that LBVs have evolved horizontally to lower effective temperatures, for which one expects an enhanced bound-free continuum opacity in the outer stellar envelope. If we include such additional opacity sources, the generalized “continuum” Eddington factor could well approach or exceed unity, at least in restricted portions of the stellar envelope. In the following, we will examine the potential consequences of exceeding such a generalized

Eddington limit for enhancing mass loss, with consideration of several specific processes like interior convection, photon “tiring”, line driving, and rotation.

2. Convective Instability and Pressure Inversion

First it should be emphasized that locally exceeding the Eddington limit does *not* necessarily lead to mass outflow. In particular, in the stellar interior $\Gamma \rightarrow 1$ generally implies through the Schwarzschild criterion that the material is convectively *unstable* (e.g., Maeder 1989). Since convection in such deep layers is highly efficient, the radiative luminosity is reduced, thereby lowering the associated radiative Eddington factor away from unity. Indeed, even in the classical case of only electron scattering opacity, the Eddington factor $\Gamma_e \sim L/M$ should increase inwards because, while the luminosity is nearly constant outside a very compact nuclear generation core, the mass within the local radius $M(r)$ decreases inward. Thus, for example, if the luminosity and surface mass M_* imply a surface Eddington factor $\Gamma_* \lesssim 1$, then the entire region interior to a mass fraction $M(r)/M_* = \Gamma_*$ should be convectively unstable!

Likewise, if an *outward* increase in the total effective opacity leads toward a superEddington condition in some region of the outer envelope, that region should also become convective. Again, as long as the convection is efficient, the net flux of radiative luminosity will be reduced so as to keep the total outward radiative force below the inward force of gravity. This suggests that a radiatively driven outflow can only be initiated *outside* the region where convection is efficient. An upper bound to the convective energy flux is set by

$$F_{conv} \approx v_{conv} l dU/dr \lesssim a H dP/dr \approx a^3 \rho, \quad (1)$$

where v_{conv} , l , and U are the convective velocity, mixing length, and internal energy density, and a , H , P , and ρ are the sound speed, pressure scale height, pressure, and mass density. Setting this maximum convective flux equal to the total stellar energy flux $L/4\pi r^2$ yields an estimate for the maximum mass loss rate that can be initiated by radiative driving,

$$\dot{M} \leq \frac{L}{a^2} \equiv \dot{M}_{max,conv}. \quad (2)$$

As we shall discuss further below, this is a very large upper limit, generally well in excess of the “photon tiring” limit set by the energy available to lift the material out of the star’s gravitational potential.

Even above the efficient-convection radius, a *limited* superEddington domain could, instead of an outflow, merely induce a pressure inversion layer (Maeder 1989), set by integrating the equation of hydrostatic equilibrium,

$$\frac{1}{\rho} \frac{dP}{dr} = \frac{GM}{r^2} (\Gamma - 1). \quad (3)$$

For example, for a narrow ($\Delta r \ll r$), isothermal, superEddington layer, the pressure would increase by a factor $\exp[(\Delta r/H)(\bar{\Gamma} - 1)]$, where $H \equiv a^2 r^2 / GM$ is the usual gravitational scale height, and $\bar{\Gamma}$ is the average of Γ over the layer. This exponential pressure increase implies, however, that such inversions are only possible over a limited domain, since eventually the star must match an outer boundary condition of negligible pressure.

3. SuperEddington Outflow and Photon “Tiring”

Let us thus next examine the possibility that such a superEddington layer does lead to sustained outflow. For simplicity, we assume the Eddington factor, $\Gamma(r)$, has a known, explicit spatial dependence, with the initial radius $r = R_*$ at which Γ first exceeds unity corresponding to the sonic point of a supersonic mass outflow. The density ρ_* and sound speed a_* at this point set the mass loss rate $\dot{M} = 4\pi R_*^2 \rho_* a_*$, but otherwise gas pressure terms have negligible effect in the further supersonic acceleration of the outflow. The steady-state equation of motion thus reduces to

$$v \frac{dv}{dr} \approx -\frac{GM_*(1 - \Gamma(r))}{r^2} \quad ; \quad r \geq R_*. \quad (4)$$

Note that in this form the mass loss rate itself has scaled out, so that the resulting velocity law would be entirely independent of the amount of mass accelerated. More realistically, a given radiative luminosity can only accelerate a limited mass loss rate before the energy expended in accelerating the outflow against gravity would necessarily come at the expense of a notable reduction in the radiative energy flux itself. To take account of this “photon tiring”, we simply reduce the radiative luminosity according to the gained kinetic and potential energy of the flow,

$$L = L_* - \dot{M} \left[\frac{v^2}{2} + \frac{GM_*}{R_*} - \frac{GM_*}{r} \right]. \quad (5)$$

Defining the scaled variables

$$w \equiv v^2 R_*/2GM_* \quad ; \quad x \equiv 1 - R_*/r, \quad (6)$$

we find the equation of motion with photon tiring can be written in the dimensionless form,

$$\frac{dw}{dx} = -1 + \Gamma(x)[1 - m(w + x)], \quad (7)$$

where the photon “tiring number”,

$$m \equiv \frac{\dot{M}GM_*}{L_*R_*} \approx 10^{-2} \frac{\dot{M}_{-4}V_{1000}^2}{L_6}, \quad (8)$$

characterizes the fraction of radiative energy lost in lifting the wind out of the stellar gravitational potential from R_* . The last expression allows easy evaluation of the likely importance of photon tiring for characteristic scalings, where $\dot{M}_{-4} \equiv \dot{M}/10^{-4} M_\odot/\text{yr}$, $L_6 \equiv L_*/10^6 L_\odot$, and $V_{1000} \equiv v_{esc}/1000 \text{ km/s} \approx 0.4 (M_*/R_*)/(M_\odot/R_\odot)$. Using integrating factors, it is possible to obtain an explicit solution to $w(x)$ in terms of the integral quantity $\bar{\Gamma}(x) \equiv \int_0^x dx' \Gamma(x')$,

$$w(x) = -x + \frac{1}{m} \left[1 - e^{-m\bar{\Gamma}(x)} \right] + w_o, \quad (9)$$

where for typical hot-star atmospheres the sonic point boundary value is very small, $w(0) = w_o \equiv a_*^2 R_*/2GM_* < 10^{-3}$.

As a simple example, consider the power law form $\Gamma(x) = 1 + 0.1\sqrt{x}$. Fig. 1a plots solutions $w(x)$ vs. x for various m . For low m , the flow reaches a finite speed at large radii ($x = 1$), but for high m , it curves back, stopping at some finite *stagnation* point x_s , where $w(x_s) \equiv 0$. The latter solutions represent flows for which the mass loss rate is too high for the given stellar luminosity to be able to lift the material to full escape at large radii. By considering the critical case $w(x = 1) = 0$, we can define a maximum mass loss rate from m_{max} , given from eqn. (9) by the transcendental relation,

$$m_{max} = 1 - e^{-m_{max}\bar{\Gamma}(1)} \approx 1 - e^{2-2\bar{\Gamma}(1)}, \quad (10)$$

where the last expression provides a good explicit approximation for any realistic $\bar{\Gamma}(1) > 1$.

Note that regardless of how large $\bar{\Gamma}(1)$ becomes, it is always true that $m_{max} < 1$, simply reflecting the fact that the mass loss is always limited by the rate at which the radiative luminosity can lift material out of the gravitational potential from R_* . By comparison, the maximum mass loss allowed by convective inefficiency (cf. eqn. (2)) would correspond to a tiring number of order $m_{conv} \approx GM/R_*a^2 \approx 2v_{esc}^2/a^2$. Thus since typically $m_{conv} \gg 1$, we conclude that any superEddington outflow initiated near the layer where convection becomes inefficient would generally stagnate by photon tiring well before any material could escape.

In the limit of negligible tiring $m \ll 1$, the flow solution (9) simplifies to

$$w(x) \approx \bar{\Gamma}(x) - x. \quad (11)$$

For a limited superEddington domain, the critical case of marginal escape with zero terminal velocity, $w(1) = 0$, is now set in general by the condition $\bar{\Gamma}(1) = 1$. Fig. 1b illustrates solutions for the specific example of nonmonotonic $\Gamma(x) = 1 + 0.1\sqrt{x} - cx$, for various c . For all $\bar{\Gamma}(1) < 1$ (i.e., $c > 0.133$), the material stagnates at the radius where $\bar{\Gamma}(x_s) = x_s$, and so cannot escape the system in a steady state flow. In a time-dependent model, such material can be expected to accumulate at this stagnation radius, and possibly eventually fall back to the star. This represents another way in which, instead of a steady outflow, a limited superEddington region could give rise to an extended envelope with either a mass circulation or a density inversion.

Of course, in realistic models the spatial variation of the radiative driving factor Γ cannot be written as such a simple, explicit function, but rather is determined implicitly through global dependence on state variables. For example, the component due to bound-free continuum Γ_{bf} depends, though the ionization dependence of the opacity κ_{bf} , on the temperature and density, as well as on a transfer solution for the radiative flux that properly accounts for ionization edges. In some circumstances, conditions may well cause the combined continuum factor $\Gamma_c = \Gamma_e + \Gamma_{bf}$ to increase outward, and perhaps even exceed unity, but the above analysis suggests that this will only lead to a sustained, continuum-driven outflow if this superEddington condition is maintained in only the outer, lower density parts of the stellar envelope. In the absence of any such ‘‘fine tuning’’, the more likely consequence will be an extended stellar envelope and/or atmosphere. Let us next consider the nature of a *line-driven* mass outflow from such an continuum-extended stellar envelope.

4. Continuum Enhanced, Line-Driven Mass Loss

For *line* opacity, the effectiveness of radiative driving can be greatly enhanced by the desaturation of the radiative flux within the line by the progressive Doppler shift from a flow *acceleration*. This characteristic leads to a natural “fine-tuning” by which the lifting of the overlying mass by a line-driven wind allows a pressure induced initial acceleration that is then further sustained by the line-force. Within the CAK formalism of a line ensemble with a power-law opacity distribution, the acceleration-dependence of the cumulative line-driving factor can be written in the form,

$$\Gamma_l = f \frac{(\bar{Q}\Gamma_\epsilon)^{1-\alpha}}{1-\alpha} \left(\frac{dw}{dx} \frac{L}{Mc^2} \right)^\alpha. \quad (12)$$

Here α is the usual CAK exponent, and \bar{Q} is a line-ensemble normalization factor related to the standard CAK constant k ($= \bar{Q}^{1-\alpha}(v_{th}/c)^\alpha/1-\alpha$), but which remains relatively fixed at $\bar{Q} \sim 10^3$ for a wide range of stellar parameters (Gayley 1995). The factor f takes account of the finite angular extent of the stellar disk (Friend & Abbott 1986).

Let us thus consider the case of a total Eddington factor $\Gamma = \Gamma_c + \Gamma_l$, with a constant $\Gamma_c \lesssim 1$. From the equation of motion (7) in the limit $m \ll 1$ of negligible photon tiring, the standard CAK formalism yields a characteristic, ‘critical’ wind solution, with terminal speed

$$v_\infty = \frac{\alpha}{1-\alpha} v_{esc} \sqrt{1-\Gamma_c} \quad (13)$$

and mass loss rate

$$\dot{M} \approx \frac{L}{c^2} \frac{\alpha/2}{1-\alpha} \left(\frac{\bar{Q}\Gamma_\epsilon}{1-\Gamma_c} \right)^{-1+1/\alpha}. \quad (14)$$

Applying $Q = 10^3$, we find the mass loss rate depends sensitively on the parameter α , with characteristic scalings,

$$\begin{aligned} \dot{M}_{-6} &\approx 2L_6 \left(\frac{\Gamma_\epsilon}{1-\Gamma_c} \right)^{\frac{1}{2}} ; \quad \alpha = 2/3 \\ &\approx 30L_6 \left(\frac{\Gamma_\epsilon}{1-\Gamma_c} \right)^1 ; \quad \alpha = 1/2. \end{aligned} \quad (15)$$

Note also that the increase in mass loss toward the Eddington limit $\Gamma_c \rightarrow 1$ is much stronger for lower values of α . In particular, photon tiring becomes significant (in the sense that $m > 0.1$) for $\Gamma_c > 0.97$ when $\alpha = 1/2$, but only for $\Gamma_c > 0.9999$ when $\alpha = 2/3$.

5. Latitudinal Mass Loss Variation in Rotating Stars

In rotating stars, the effective stellar gravity is further reduced by the centrifugal force,

$$g_{eff}(\theta) = \frac{GM}{R^2} (1 - \Gamma_c - \Omega \sin \theta), \quad (16)$$

where θ is the stellar colatitude, $\Omega \equiv v_{rot}^2 R/GM$, and v_{rot} is the surface rotation speed. Using this to generalize the mass loss scaling in eqn. (14), we derive for the latitudinal dependence of the mass flux,

$$\dot{m}(\theta) \sim F(\theta)^{1/\alpha} g_{eff}(\theta)^{1-1/\alpha}, \quad (17)$$

where $F(\theta)$ gives the latitudinal dependence of the radiative flux. For example, for $\alpha = 1/2$, we find the scaling

$$\begin{aligned} \dot{m}(\theta) &\sim \frac{1}{g_{eff}(\theta)} ; F(\theta) = const. \\ &\sim g_{eff}(\theta) ; F(\theta) \sim g_{eff}(\theta). \end{aligned} \quad (18)$$

The first form corresponds to the original result of Friend & Abbott (1986), who implicitly assumed the radiative flux is constant in latitude, and so concluded that the mass flux, and thus density, would be maximum near the stellar equator, where the effective gravity is most reduced by the centrifugal force of the stellar rotation. By contrast, the latter form assumes a standard von Zeipel (1924) *gravity-darkening law* in which the radiative flux itself scales in proportion to the effective surface gravity. As recently pointed out by Owocki, Cranmer, & Gayley (1996), this leads to a quite different, somewhat surprising result, namely that the mass flux should scale *directly* with effective gravity, and thus be at a *minimum* near the equator!

Owocki et al. (1996) also point out the likely importance of *nonradial* components of the line-force in directing material away from the equator. Their 2-D simulations of rotating, line-driven winds show how these can effectively inhibit the formation of the equatorial *Wind Compressed Disk* (WCD) predicted for rapidly rotating outflows driven by strictly radial forces (Bjorkman & Cassinelli 1993; Owocki et al. 1994). At present, these results should perhaps be viewed as a curious puzzle, since they apparently run contrary to the general empirical evidence for enhanced density and mass flux in the equatorial regions around rapidly rotating early-type stars (see, e.g., introduction from Bjorkman & Cassinelli 1993). So far the models have not considered the possible effect of a latitudinal variation in the line-driving parameters (e.g. α), such as proposed by Lamers & Pauldrach (1991) in their “bi-stability” model for B[e] stars. Furthermore, though the theoretical prediction of substantial gravity darkening is quite clear for the radiative envelopes expected in early-type stars, there is little direct observational data on how much equatorial darkening does in fact occur.

Notwithstanding these caveats, it is interesting to consider the contrast between the outflow geometries predicted by wind models with or without gravity darkening. As one nears what N. Langer (these proceedings) has termed the “Omega limit” $\Omega \rightarrow 1 - \Gamma_c$, both models predict a sharp increase in the total mass loss integrated over the entire surface. However, the model without gravity darkening predicts this increased mass loss will be concentrated near the equator, while the model with gravity darkening has it occurring near the pole! The shape of the resulting wind-blown nebula should be oblate in the former case, and *prolate* in the latter. Both types may occur in observed nebulae from known and suspected LBVs, but certainly for the most spectacular example, the homonculus in η Carina, most of the mass appears contained in the bipolar

outflow, and thus is manifestly prolate. There are alternative explanations for this apparent geometry (e.g. magnetic confinement, or spherical expansion into a preexisting WCD, cf. Garcia-Segura, these proceedings), but one possibility to consider is that this could reflect an equatorial-darkened radiation field during the outburst.

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